APPENDIX D

THE USE OF IMPERIAL SYSTEM (FPS) UNITS

APPENDIX D

THE USE OF IMPERIAL SYSTEM (FPS) UNITS

A great deal of confusion arises with the use of English units or the FPS (foot/pound/second) system. The system is intended to be user friendly and in many ways it is. For example, 1 pound mass will produce 1 pound force in Earth's gravitational field. Also 1 inch is about the size of the thumb from the last joint to the tip of the nail. Also a foot is about the size of an average man's foot, etc. The price to pay for this simplicity is that a conversion constant must be used to make the units consistent. A case in point is Newton's second law, which expresses the relationship between force and acceleration.

Newton's second law of motion is:

$$F = ma \qquad \text{[D-1]}$$

where *F* is a force, *m* the mass of the object, and *a* the acceleration of the object.

The weight of an object, or the force produced by an object with respect to the surface of another object (the surface of a scale for example) under the Earth's constant gravitational field is proportional to it's mass times the gravitational acceleration.

$$a = g \quad therefore \quad F = mg$$

"g" has a value of 32.17 ft/s^2 on average. To have 1 pound mass produce 1 pound force, the constant g_c must be used:

$$g_c = 32.17 \frac{lbm - ft}{lbf - s^2}$$

We introduce the constant g_c into Newton's second law to provide the units of the Imperial system:

$$F(lbf) = m(lbm) \frac{g(ft/s^2)}{g_c\left(\frac{lbm - ft}{lbf - s^2}\right)} \qquad \text{[D-2]}$$

Some of the units on the right hand side of equation [D-2] cancel out and the resultant combined unit is the pound-force (lbf-ft) a unit of energy.

On the surface of the Earth 1 pound mass produces 1 pound force. On the surface of the moon, where the acceleration of the gravitational field is 1/6 that of the Earth's, 1 pound mass will produce 1/6 pound force.

$$F(lbf)=m(lbm)\frac{\frac{32.17}{6}\left(ft/s2\right)}{32.17\left(\frac{lbm-ft}{lbf-s2}\right)}=\frac{1}{6}m$$

The metric system or SI system is designed to be consistent with Newton's second law without the use of any conversion constant. In the metric system, one kilogram of mass on the Earth's surface produces 9.8 Newtons of force. The acceleration g produced by Earth's gravitational field is 9.8 m/s^2.

$$F = mg$$

$$F(N)=m(kg)9.8\left(\frac{m}{s2}\right)= 9.8\frac{kg-m}{s^2}$$

therefore 1 kilogram (kg) of mass produces 9.8 Newtons (N). A Newton by definition equals 1 kg-m/s^2.

Kinetic energy in FPS units

Kinetic energy is proportional to mass, times velocity squared:

$$KE = \frac{1}{2}mv^2 \qquad \text{[D-3]}$$

A typical energy unit in the FPS system is the lbf-ft. To obtain lbf-ft in the above equation we use the constant g_c. The value of g_c is 32.17 lbm-ft/lbf-s^2.

$$KE(lbf-ft)=\frac{1}{2}\frac{m(lbm)v^2(ft/s)^2}{g_c\left(\frac{lbm-ft}{lbf-s^2}\right)} \qquad \text{[D-4]}$$

Some of the units on the right hand side of equation [D-4] cancel out and the resultant combined unit is the pound-force (lbf-ft) a unit of energy.

Potential energy in FPS units

Potential energy is equal to the weight of the object (mg) times its elevation (z) above as reference plane.

$$PE = mgz \qquad \text{[D-5]}$$

Again the typical energy unit in the FPS system is the lbf-ft. To obtain lbf-ft in the above equation we use the constant g_c.

$$PE(lbf-ft) = \frac{m(lbm)\ \ g(ft/s^2)z(ft)}{g_c\left(\frac{lbm-ft}{lbf-s^2}\right)} \qquad \text{[D-6]}$$

Some of the units on the right hand side of equation [D-6] cancel out and the resultant combined unit is the pound-force (lbf-ft) a unit of energy.

The conversion of head to pressure in the FPS system

Pressure is equal to head, times the specific weight or head, times the density times the acceleration due to gravity.

$$p = \gamma\ H = \rho\ g\ H \qquad \text{[D-7]}$$

where p (lbf/ft^2) is pressure, ρ (lbf/ft^3) is the density, H(ft) is head and g is the acceleration due to gravity. To obtain lbf/ft^2 in the above equation we use the constant g_c.

$$p(lbf/ft^2) = \frac{\rho(lbm/ft^3)\ g(ft/s^2)\ H(ft)}{g_c\left(\frac{lbm-ft}{lbf-s^2}\right)} \qquad \text{[D-8]}$$

Some of the units on the right hand side of equation [D-8] cancel out and the resultant combined unit is the pound per square foot (lbf/ft^2) a unit of pressure.