APPENDIX B

THE NEWTON-RAPHSON ITERATION TECHNIQUE APPLIED TO THE
COLEBROOK EQUATION

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THE NEWTON-RAPHSON ITERATION TECHNIQUE

Since the value for *f* in the Colebrook equation cannot be explicitly extracted from the equation, a numerical method is required to find the solution. Like all numerical methods, we first assume a value for *f*, and then, in successive calculations, bring the original assumption closer to the true value. Depending on the technique used, this can be a long or slow process. The Newton-Raphson method has the advantage of converging very rapidly to a precise solution. Normally only two or three iterations are required.

The Colebrook equation is:

$$\frac{1}{\sqrt{f}} = -2 \, \log_{10}\left(\frac{\varepsilon}{3.7\,D} + \frac{2.51}{R_e \, \sqrt{f}}\right)$$

The technique can be summarized as follows:

1. Re-write the Colebrook equation as:

$$F = \frac{1}{\sqrt{f}} + 2 \, \log_{10}\left(\frac{\varepsilon}{3.7\,D} + \frac{2.51}{R_e \, \sqrt{f}}\right) = 0$$

2. Take the derivative of the function *F* with respect to *f*:

$$\frac{dF}{df} = -\frac{1}{2}\,f^{-3/2}\left(1 + \frac{2.51\,\log_{10} e}{\left(\frac{\varepsilon}{3.7\,D} + \frac{2.52\,f^{-1/2}}{Re}\right)Re}\right)$$

3. Give a trial value to *f*. The function *F* will have a residue (a non-zero value). This residue (RES) will tend towards zero very rapidly if we use the derivative of *F* in the calculation of the residue.

$$f_n = f_{n-1} - RES \quad with \quad RES = \frac{F}{\dfrac{dF}{df}}$$

For *n = 0* assume a value for f_0, calculate *RES* and then f_1, repeat the process until *RES* is sufficiently small (for example *RES* < 1 x 10^{-6}).