APPENDIX A

USEFUL EQUATIONS (METRIC AND IMPERIAL SYSTEMS)

THE DEFINITION OF VISCOSITY

RHEOLOGICAL (VISCOUS BEHAVIOR) PROPERTIES OF FLUIDS

APPENDIX A

USEFUL EQUATIONS		
	FPS Units	SI Units
Flow vs. velocity	$v(ft/s) = 0.4085 \dfrac{q(USgal./\min)}{D^2(in)^2}$	$v(m/s) = 21.22 \dfrac{q(L/\min)}{(D(mm))^2}$
Specific gravity vs. fluid density	$SG = \dfrac{\rho_F}{\rho_W}$ where ρ_W = 62.34 lbm/ft^3 for water at 60 °F	$SG = \dfrac{\rho_F}{\rho_W}$ where ρ_W = 997.8 kg/m^3 for water at 15.55 °C
Kinematic viscosity ν vs. dynamic (absolute) viscosity μ	$v(cSt) = 62.45 \times \dfrac{\mu(cP)}{\rho\left(\dfrac{lbm}{ft^3}\right)}$	$v(cSt) = 10^3 \times \dfrac{\mu(cP)}{\rho\left(\dfrac{kg}{m^3}\right)}$
Kinematic viscosity vs. dynamic (absolute) viscosity	$v(SSU) = 4.63 \dfrac{\mu(cP)}{SG}$ where ρ_W = 62.34 lbm/ft^3 for water at 60 °F	$v(cSt) = 1.0022 \dfrac{\mu(cP)}{SG}$ where ρ_W = 997.8 kg/m^3 for water at 15.55 °C
Kinematic viscosity: SSU vs. cSt	$v(SSU) = v(cSt) \times 4.635 \; for \; v(cSt) > 50$	
Pressure vs. pressure head or fluid column height	$H_f(ft\ fluid) = 2.31 \dfrac{p(psi)}{SG}$ based on ρ_W = 62.34 lbm/ft^3 for water at 60 °F	$H_f(m\ fluid) = 0.102 \dfrac{p(kPa)}{SG}$ based on ρ_W = 997.8 kg/m^3 for water at 15.55 °C
Reynolds number	$R_e = 7745.8 \dfrac{v(ft/s)D(in)}{v(cSt)}$	$R_e = 1000 \dfrac{v(m/s)D(mm)}{v(cSt)}$
Pipe friction loss	$\dfrac{\Delta H_{FP}}{L}\left(\dfrac{ft\ fluid}{100ft\ of\ pipe}\right) = 1200 f \dfrac{(v(ft/s))^2}{D(in) \times 2g(ft/s^2)}$ The Darcy-Weisbach equation g=32.17 ft/s^2	FPS units
Pipe friction loss	$\dfrac{\Delta H_{FP}}{L}\left(\dfrac{m\ fluid}{100\ m of\ pipe}\right) = 10^5\ f\ \dfrac{(v(m/s))^2}{D\ (mm)\ \times\ 2g\ (m/s^2)}$ units The Darcy-Weisbach equation g= 9.81 m/s^2	SI
Fittings friction loss	$\Delta H_{FF}(ft\ fluid) = K \dfrac{v^2(ft/s)^2}{2g\ (ft/s^2)}$	$\Delta H_{FF}(m\ fluid) = K \dfrac{v^2(m/s)^2}{2g\ (m/s^2)}$
Friction parameter for the laminar flow regime	$f = \dfrac{64}{R_e}$ *f* and *Re* are non-dimensional units.	
Friction parameter for the turbulent flow regime	$\dfrac{1}{\sqrt{f}} = -2\ \log_{10}\left(\dfrac{\varepsilon}{3.7D} + \dfrac{2.51}{R_e\sqrt{f}}\right)$ The Colebrook equation	

	FPS Units	SI Units
Friction parameter for the turbulent flow regime	$f = \dfrac{0.25}{\left(\log_{10}\left(\dfrac{\varepsilon}{3.7\,D} + \dfrac{5.74}{R_e^{\,0.9}}\right)\right)^2}$ The Swamee & Jain equation. Can be used as a replacement for the Colebrook equation.	
Pipe friction loss	$\dfrac{\Delta H_{FP}}{L}\left(\dfrac{ft \quad fluid}{100ft \quad of\,pipe}\right) = 0.20083 \times \left(\dfrac{100}{C} \times q(USgpm)\right)^{1.85} \times \left(\dfrac{1}{D(in)}\right)^{4.8655}$ FPS units The Hazen-Williams equation	
Valve CV coefficient	$CV\left(\dfrac{USgpm}{psi^{1/2}}\right) = \dfrac{q\,(USgpm)}{\sqrt{\dfrac{\Delta p\,(psi)}{SG}}}$	FPS units
Total Head	$\Delta H_P = \Delta H_{F1-2} + \Delta H_{EQ1-2} + \dfrac{1}{2g}(v_2^{\,2} - v_1^{\,2}) + z_2 + H_2 - (z_1 + H_1)$ see Figure A-1, the units are in feet or meters of fluid	
N.P.S.H. available	$N.P.S.H._{avail} = -(\Delta H_{F1-S} + \Delta H_{EQ1-S}) + \dfrac{v_1^{\,2}}{2g} + (z_1 - z_S + H_1) + H_B - H_{va}$ see Figure A-1, the units are in feet or meters of fluid	
Pump shaft brake power	$P(hp) = \dfrac{SG\,\Delta H_P(ft\ fluid)\,q(USgal/\text{min})}{3960\,\eta}$	$P(kW) = \dfrac{SG\,\Delta H_P\ (m\ fluid)\ q(L/\text{min})}{6128\,\eta}$
	SLURRY CALCULATIONS	
Mass flow rate of a slurry or mixture of solid particles in a liquid medium	$M\left(\dfrac{tn}{h}\right) = 0.25 \times q\left(\dfrac{USgals.}{\text{min}}\right) \times C_V \times SG_S$ tn: tonnes or 2000 pound mass	$M\left(\dfrac{t}{h}\right) = 0.06 \times q\left(\dfrac{L}{\text{min}}\right) \times C_V \times SG_S$ t: metric tonne or 1000 kg
Concentration ratio by weight of a slurry	$C_W = \dfrac{C_V \times SG_S}{SG_M}$	
Concentration ratio by volume of a slurry	$C_V = \left(\dfrac{1}{C_W\left(\dfrac{1}{SG_S} - 1\right) + 1} - 1\right) \times \dfrac{1}{(SG_S - 1)}$	
Specific gravity of slurry	$SG_M = SG_L + C_V(SG_S - SG_L)$ SG_L is the specific gravity of the transport liquid, for water SG_L = 1.0.	
	PULP SLURRY	
Mass flow rate of a pulp suspension	$M\left(\dfrac{tn}{day}\right) = 0.06 \times q\left(\dfrac{USgals.}{\text{min}}\right) \times \%C_V$ For pulp slurries, SG_M = 1	$M\left(\dfrac{t}{day}\right) = 0.0144 \times q\left(\dfrac{L}{\text{min}}\right) \times \%C_V$

Table A-1 Useful Equations

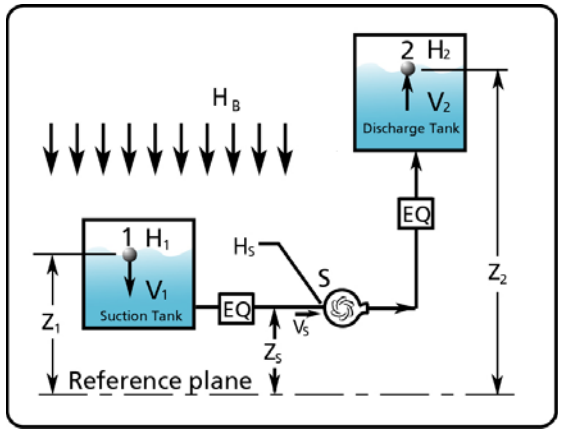


Figure A-1 Typical pump system.

THE DEFINITION OF VISCOSITY

Isaac Newton investigated the fluid property called viscosity. He set up an experiment designed to measure viscosity. The experiment consisted in the interaction of two plates separated by a short distance, one fixed and one moving with fluid between. He theorized that it required a force (F) to move the top plate at a given velocity, v. If the top plate was moving at velocity v, he deduced that the fluid layer just underneath the plate must be moving at the same velocity or else the plate would be skipping over the fluid surface. In addition, force (F) can only be produced if the top layer of fluid is attached to the top plate and the bottom layer to the bottom plate. Since the bottom plate is not moving, the bottom fluid layer is also not moving, or has zero velocity. Therefore, the fluid layers between the moving surface and the fixed one have different velocities. Each layer of fluid is traveling at a different speed. It is this speed variation (or velocity gradient (dv/dy), see Figure A-2) that is the cause of viscosity and is responsible for shearing the fluid internally. Newton's assumption was that the velocity gradient is independent of viscosity. In other words, a force twice as large would be required to move the fluid twice as fast, meaning there must be a constant relationship between the force (F) required to move the fluid and the rate of shear. Alternatively, in mathematical form, the force (F) is proportional to the velocity gradient (dv/dy) :

$$F = K\frac{dv}{dy} \text{ or } \tau = \frac{K}{A}\frac{dv}{dy} = \frac{1}{\frac{A}{K}}\frac{dv}{dy} = \frac{1}{\mu}\frac{dv}{dy} \tag{A-1}$$

The constant (A/K) is called the viscosity of the fluid and is represented by the Greek letter μ (mu). The value of μ will determine the magnitude of the shearing force (F). Fluids with higher viscosities will require a greater shearing force for the same velocity differential. Since the experiment should be valid for fluid bodies of any size, the tangential stress (τ = F/A) is a more appropriate parameter to relate to viscosity.

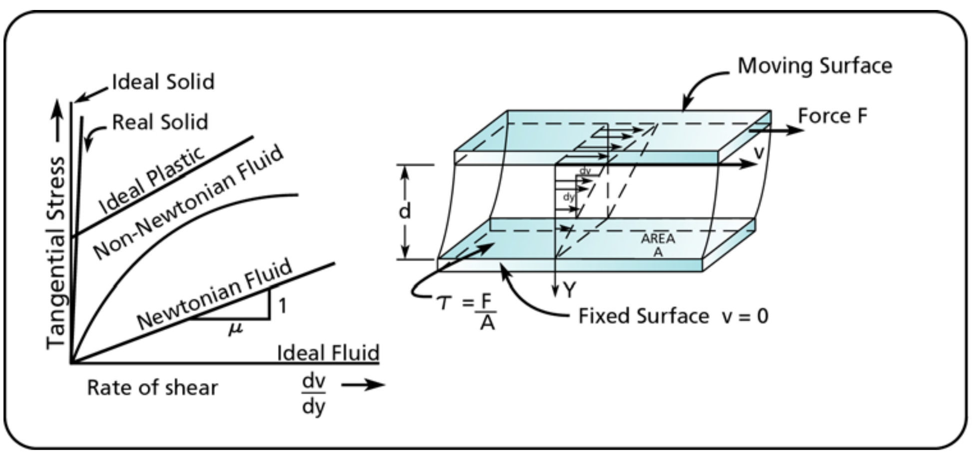


Figure A-2 Viscosity vs. the velocity gradient and the tangential stress.

The term μ is known as the absolute viscosity of the fluid (see equation [A-1]). The velocity gradient *dv/dy* is known as the rate of shear. Newton could not test his hypothesis because of experimental difficulties. Many years later, Poiseuille (1849) developed an experimental method that consisted in measuring the flow of liquid in a small tube and relating the pressure driving the fluid through the end of the tube to the flow and viscosity. Poiseuille's experimental apparatus verified the correctness of Newton's hypothesis. Newton's viscosity equation describes a class of fluids that came to be known as Newtonian fluids. Many fluids behave in this fashion (see Table A-2). The unit of absolute viscosity is the Poise (or centiPoise), in honor of Poiseuille. One (1) centiPoise (the unit symbol is cP) is the viscosity of water at 68 ^{0}F, making it easy to compare the viscosity of various fluids to that of water.

Many fluids do not behave in the well-ordered fashion of Newtonian fluids. These are known as non-Newtonian fluids and fall in several categories (see Table A-2) depending on what shape the tangential stress vs. velocity gradient takes. For these fluids, the viscosity is variable. In the literature, a variable viscosity is often referred to as apparent viscosity. The velocity gradient affects the viscosity, resulting in a much higher (or in some cases lower) tangential stress than for a Newtonian fluid.

A typical household product will help illustrate this point. Try the following experiment. In a large shallow bowl make a solution of approximately 1 part water and 2 parts cornstarch, try moving this fluid rapidly around with your fingers. When the fingers are moved slowly, the solution behaves as expected, offering little resistance. The faster you try to move through the fluid, the higher the resistance. At that rate of shear, the solution almost behaves as a solid, If you move your fingers fast enough they will skip over the surface. This is what is meant by viscosity being dependent on rate of shear. Compare this behavior to that of molasses; you will find that even though molasses is viscous its viscosity changes very little with the shear rate. Molasses flows readily no matter how fast the movement.

This explains why centrifugal pumps with their high rate of shear are not suitable for non-Newtonian fluids. A pump of the fixed displacement type, operating at low speed, is more appropriate.

Kinematic viscosity

A term frequently used to represent viscosity (for example in the definition of the Reynolds number) is the kinematic viscosity ν (nu). The relationship between the absolute and kinematic viscosity is:

$$\nu = \frac{\mu}{\rho}$$

The kinematic viscosity of water at 68 ^{0}F is 1 centiStoke (cSt), and was named in honor of G.G. Stokes of the Navier-Stokes equation fame.

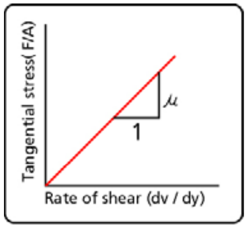
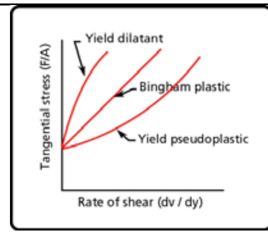
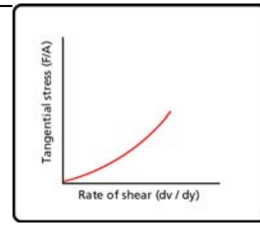
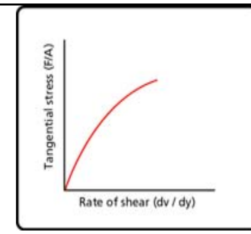
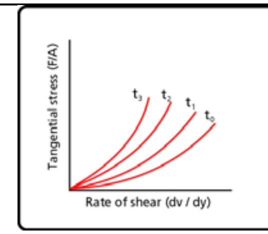
NEWTONIAN	NON NEWTONIAN				
Newtonian	Non Newtonian	Bingham plastic Yield pseudoplastic Yield dilatant	Pseudoplastic	Dilatant	Thixotropic Rheopectic
					
			Viscosity increases slowly with the rate of shear.	Viscosity increases with the rate of shear.	Thix.: viscosity decrease with time. Rheop.: viscosity increase with time.
- water - high viscosity fuel - some motor oils - most mineral oils - gasoline - kerosene - most salt solutions in water - light suspensions of dye stuff - kaolin (clay slurry)	oils containing polymeric thickeners, viscosity index improvers and waxy or soot particles	- thermoplastic polymer solutions - sewage sludge's - digested sewage - clay - mud - ketchup - chewing gum - tar - high concentrations of asbestine in oil	- GRS latex solutions - sewage sludge's - grease - molasses - paint - starch - soap - most emulsions - printer's ink - paper pulp	- starch in water - beach sand - quicksand - feldspar - mica - clay - candy compounds - peanut butter	- most paints (thixo.) - silica gel - greases - inks - milk - mayonnaise - carboxymethyl cellulose - bentonite (rheop.) - gypsum in water - asphalt - glues - molasses - starch - lard - fruit juice concentrates

Table A-2 Rheological properties of fluids (see references 2, 6, 12 and 13)