COMPARATIVE STABILITY ANALYSIS OF TAILINGS STORAGE FACILITIES

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ABSTRACT

Tailings Storage Facilities (TSFs) are vast structures that respond to site-specific characteristics, rate of production, placement techniques, and the physical and mechanical properties of tailings. As a common denominator, TSFs are in some degree vulnerable to failure due to liquefaction, poor management, slope instability and/or unusual climatic events. In practice, stability of TSFS is determined by calculating a minimum Factor of Safety (FOS) using the Limit Equilibrium Method (LEM). However, it has been proven that relying exclusively on the limit equilibrium approach is not accurate because it is basically a static method that does not take into account the stress-strain distribution and displacement experienced by the constitutive materials of a TSF. In order to overcome these limitations of the LEM, the finite element method - Shear Reduction Technique (SRT) has been used as a more reliable tool for TSFs stability assessment. This paper presents a comparative stability analysis between an upstream tailings storage facility (UTSF), and a water retention tailings dam (WRTD) under static and pseudo-static states using simplified and rigorous LEMs and the SRT. Additionally, and taking into consideration the intrinsic uncertainty of tailings properties, sensitivity and probabilistic analyses in the form of Monte Carlo Simulation (MCS) and the Point Estimate Method (PEM) are applied to determine the Probability of Failure *(Pf)* and Reliability Index *(β)* of each TSF. It is shown that the friction angle of UTSF's foundation and the core's cohesion of the WRTD are the main variables that govern stability. Further analysis shows that the WRTD has a higher reliability index and a lower probability of failure than the UTSF.

KEYWORDS

Tailings storage facilities (TSFs), Factor of Safety (FOS), Shear Reduction Technique (SRT), Probability of failure (*Pf*), and Reliability index (β)

INTRODUCTION

Tailings Storage Facilities (TSFs) are vast structures required for the management of milling and mineral processing wastes. TSFs design is based on site-specific variables that depend on the intrinsic properties of tailings, the project economics; the availability of embankment construction materials, water retention requirements, the regional seismicity, the environmental conditions, and the regulations applicable to the mining project.

TSFs are classified as water retention tailings dams (WRTDs) and raised embankment TSFs according to the sequence of construction, internal structure, and constitutive materials in the retaining embankment. In WRTDs, the embankment is built to its full height prior to the beginning of operations, they have an impervious core, and borrow material is used for constructing the embankment (Vick 1983). Raised embankments TSFs, are built in stages as the mining operation progresses and mill tailings are used as construction materials. In the upstream method, for example, a borrow starter dyke is initially constructed and the subsequent raising of the embankment is done using hydraulically deposited or cycloned tailings. The upstream technique is the most economical but the least favorable raised embankment method because steady state is reached only at closure. Furthermore, UTSFs have poor water storage capacity and seismic resistance (Julien & Kissiova, 2011; Qiu & Sego, 1998; Vick, 1983).

The USCOLD (1994) reports showed that UTSFs have accumulated the highest number of failure incidents compared to WRTDs and other types of raised embankment TSFs. Amongst the causes that have been attributed to UTSFs failure is excessive embankment height combined with slope instability. Likewise, a high raising rate usually translates into high pore pressure zones unevenly distributed within the impoundment, significant reduction in the shear strength of materials, and static liquefaction.

In order to assess the effect of embankment height on slope stability, this paper presents a stability analysis of an UTSF and a WRTD following a deterministic and a probabilistic approach under static and pseudo-static states. The purpose is to identify the key factors that govern the performance and reliability of each type of impoundment and to analyze the variables that require particular attention to prevent, control, and/or mitigate poor geotechnical performance.

TSFs STABILITY ANALYSES

Deterministic Approach

Generally, TSFs stability analyses are conducted by calculating a deterministic FOS using simplified and/or rigorous LEMs. However, it has been proven that relying exclusively on LEMs is inaccurate because these methods only satisfy equations of statics and do not consider the strain-stress deformation and displacement that take place in the impoundment (Hamade et al., 2011; Rocscience, 2007). The Morgenstern-Price LEM, for example, takes into account the sum of moment equilibrium, force equilibrium, and the interslice force, whereas the Ordinary-Fellenius LEM only considers moment equilibrium and no inter-slice force. The finite element - Shear Reduction Technique (SRT) is an alternative analysis method for more dvanced slope stability analyses in which the material effective cohesion (c´) and friction angle (ϕ´) are progressively reduced by a Shear Reduction Factor (SRF) until the model does not converge to a solution and equilibrium cannot be maintained.

$$\tau/SRF=c'/SRF+ \tan \phi'/SRF \tag{1}$$

$$\tau/SRF=c^*+ \tan \phi^* \tag{2}$$

$$c^*=c'/SRF \tag{3}$$

$$\phi^*=\tan^{-1}((\tan \phi')/SRF) \tag{4}$$

where c* and ϕ* are the effective and reduced Mohr-Coulomb cohesion and friction angle parameters (Rocscience, 2007). The critical factor at which failure occurs is the equivalent of the LEM-FOS. The SRT satisfies the main limitations of the LEM, that is, stress-strain and displacement are computed parallel to the FOS. Additionally, assumptions about the interslice forces, location, or shape of the sliding surface are eliminated (Duncan, 1996; Rocscience, 2007).

Probabilistic Approach

A shared limitation of both the deterministic LEM and SRT methods is that the spatial and/or temporal uncertainties associated with the physical and mechanical properties of the constitutive materials of a TSF are neglected in standard calculations. As a result, the reliability analysis of slope stability must be integrated in slope stability assessment. Studies by Hamade et al. (2011), Wang et al. (2011), Griffiths et al. (2009), and Peterson (1999) present case studies of stability analysis integrated to various probabilistic methods such as First Order Second Moment Method (FOSM), Monte Carlo Simulation (MCS), and Point Estimate Method (PEM) for reliability assessment.

In the PEM, for example, the deterministic value of the most critical random variable is replaced by a set of discrete points located plus or minus one standard deviation from the mean value. All possible solutions are calculated according to the 2^n condition, in which n is the number of random variables.

The reliability criteria commonly used to assess the stability of TSFs are the FOS, the probability of failure *(Pf)* and the Reliability Index *(β)*. The FOS should be equal to or greater than 1.3 for steady state (CDA, 2007), 1.5 for long term analysis, and 1.1 for pseudo-static analyses (CDA, 2007; MDDEP, 2012). Failure will occur when the structure's minimum FOS is less than 1 (CDA 2007; MDDEP, 2012). According to the U.S. Army Corps of Engineers, (1997) the level of performance of the TSF can be considered good if the reliability index *(β)* is greater than 3.

MODEL DESCRIPTION AND DESIGN CRITERIA

The stability analyses presented in this paper are based on the case studies presented by Saad and Mitri (2011) and Hamade el al. (2011) for the UTSF and the WRTD, respectively. All materials are assumed as elastic-perfectly plastic and following the Mohr-Coulomb failure criteria. The pseudo-static analysis is conducted assuming a horizontal seismic coefficient 0.05 and a return period of 2% based on Earthquakes Canada (2010). The stability analyses are conducted using the Effective Stress Analysis (ESA) approach because the main purpose is to observe the long-term effect of the embankment height on stability when steady-state and drained conditions are assumed.

The Upstream Tailings Storage Facility (UTSF)

The UTSF was built in seven stages. The first stage was the starter dyke and the second stage was the first tailings deposition. After, the first tailings deposition, five embankments were constructed at a raising rate of 5.25m/year. The embankment reached an ultimate height of 41.75 m as shown in Figure 1.

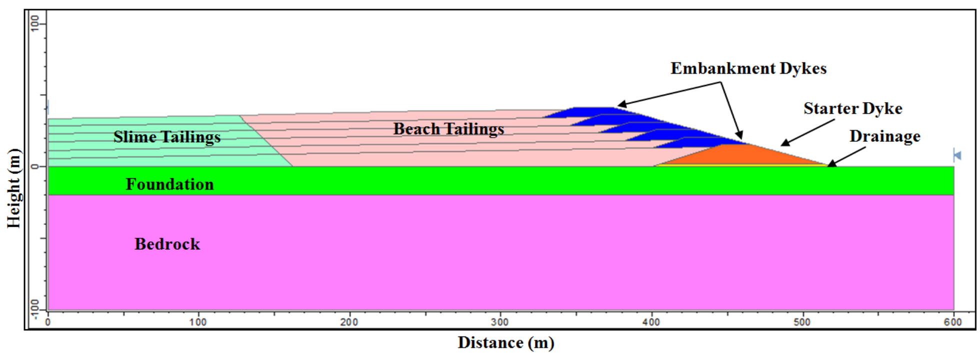


Figure 1 – Numerical model: UTSF: End of construction

The upstream slope is 3H: 1V and the downstream slope is 3.5H: 1V. The beach width at the end of construction is 162 m and has a slope of 2%. The freeboard maintained during raising and at the end of construction is 2 m. Tailings are considered fully saturated upon deposition. The minimum design distance between the embankment crest and beach phreatic surface is 50 m. The material properties of the UTSF are presented in Table 1.

Table 1 – Material properties UTSF. Adapted from Saad and Mitri (2011)

Material	γ_{sat} (kN/m^3)	γ_{dry} (kN/m^3)	ϕ' (deg)	c′ (kPa)	E (kPa)	ν	k (m/s)
Bedrock	26	25.8	42	6000	$1x10^7$	0.23	$1x10^{-8}$
Top Foundation	17.9	15.9	21	0	25000	0.2	$1x10^{-7}$
Drainage	19.4	15.5	34	0	$1x10^7$	0.28	0.06
Starter Dyke	18.5	13.6	35	0	$1x10^6$	0.3	0.001
Beach Tailings	20	14.7	30	0	5575	0.33	$7.3 x10^{-7}$
Slime Tailings	19.7	14.5	5	5	5575	0.33	$7.3x10^{-8}$
Embankment Dykes	22.3	17.9	30	0	5575	0.33	0.000115

Note: γ=unit weight; ϕ= angle of friction; c= cohesion; E= Young's modulus; ν= Poisson's ratio; k=permeability

The Water Retention Tailings Dam (WRTD)

The WRTD (Figure 2) was built to its complete height (16m) prior to the beginning of operations. The upstream slope is 2.5H: 1V and the downstream slope is 2.5H: 1V. Tailings reached an ultimate height of 18 m due to the construction of a small dyke near the end of operations. The freeboard at the end of construction is 2 m. Tailings are considered fully saturated upon deposition. The material properties of the WRTD are presented in Table 2.

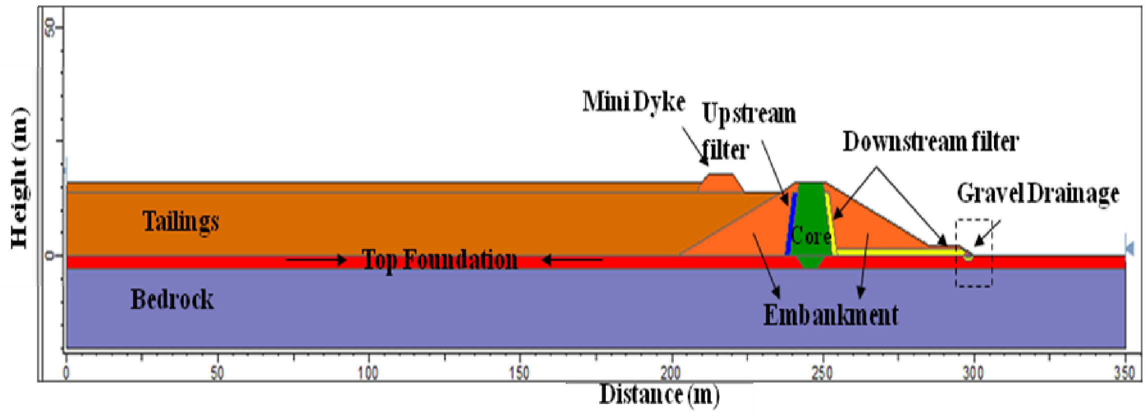


Figure 2 – Numerical model: WRTD - End of construction

Table 2 – Material properties WRTD. From Hamade et al. (2011)

Material	γ_{sat} (kN/m^3)	ϕ' (deg)	c' (kPa)	E (kPa)	ν	k (m/s)
Bedrock	27	42	6000	$2.84x10^{7}$	0.23	$1x10^{-8}$
Top Foundation	16.5	3	50	20000	0.25	$1x10^{-7}$
Core	21.5	28	12	150000	0.2	$1x10^{-7}$
Embankment	18.5	35	0	$1x10^{7}$	0.3	0.001
Downstream filter	18	34	0	70000	0.35	0.00025
Upstream filter	20	36	0	60000	0.28	0.06
Gravel Drainage	19	37	0	80000	0.3	0.1
Tailings (mill)	16	28	0	5575	0.33	$1x10^{-7}$

Note: γ=unit weight; ϕ= angle of friction; c= cohesion; E= Young's modulus; ν= Poisson's ratio; k=permeability

ANALYSIS PROCEDURE

First, static and pseudo-static stability analyses of the TSFs are conducted. Steady-state and drained conditions are assumed at each stage of construction and the FOS is calculated through the rigorous GLE-Morgenstern-Price and the simplified Ordinary-Fellenius limit equilibrium methods using the software SLIDE 6.0 by Rocscience (2006). The two-dimensional finite element SRT static and pseudo-static stability analyses are performed with the software Phase^2 by Rocscience (2007). From these analyses, the deterministic static and pseudo-static FOS of each TSF are obtained. Second, the static and pseudo-static probabilistic analyses are performed. The mean value (μ), standard deviation (σ), and coefficient of variation (COV) of the constitutive materials parameters are defined using Eq. 5.

$$COV = \frac{\sigma}{\mu} x100\% \tag{5}$$

Then, sensitivity analyses were conducted to define the critical random variables (R.V) of each TSF. The sensitivity analyses are conducted by calculating the relative minimum and relative maximum values of the unit weight, cohesion and friction angle of each material at plus or minus 3-standards-deviations-distance from the mean value (for a sample space of 99.74%). A COV of 25% was assumed (Hamade et al., 2011). From these analyses, the R.V were obtained and assigned a probabilistic distribution. After, 1000 Monte Carlo Simulation runs are performed to recalculate the probabilistic FOS, the probability of failure *(Pf)* and reliability indices in the static and pseudo static state at the last stage of construction. The PEM is used to conduct the probabilistic analyses of the SRT. The analysis procedure is summarized in Table 3.

Table 3 – Summary of analysis procedure

Analysis	Method	Modelling Tool	Procedure
Deterministic Static analysis with steady state seepage	LEMs: M-P-O-F	SLIDE 6.0	Input: Increasing embankment/tailings height Output: LEM—FOS, SRT—FOS, deformation, and displacement contours.
	FEM -SRT	PHASE^2	
Deterministic Pseudo-static analysis with steady state seepage	LEMs: M-P-O-F	SLIDE 6.0	Input: Seismic coefficient $K_{h=}$ 0.05 for the same loading criteria than static analysis. Output: pseudo static LEM—FOS, pseudo-static SRT—FOS, deformation, and displacement contours.
	FEM -SRT	PHASE^2	
Sensitivity Analysis	LEM: M-P	SLIDE 6.0 and Excel spreadsheets	Input: Mean (μ), std. dev (σ), and COV of materials properties. Relative minimum and maximum values with μ±3σ rule to cover 99.74% of sample space. Output: Random Variables of TSFs.
Static and pseudo-static analyses with seepage	1000 MCS using LEM: M-P	SLIDE 6.0 and Spreadsheets	Input: Same loading criteria than for deterministic analyses. Output: Probabilistic FOS; PDF *Pf* and *β*.
Static and pseudo-static analysis with seepage	FEM-SRT-PEM	PHASE^2	Input: Same loading criteria than deterministic pseudo-static analyses. Output: Probabilistic FOS; Solutions=2^n; n=# of R.V; *Pf* and *β*

RESULTS AND DISCUSSION

The results of the static and pseudo static deterministic LEM and SRT analyses of the UTSF and WRTD are presented in Tables 4 and 5, respectively.

Table 4 – Static and pseudo-static deterministic FOS for the UTSF using LEMs and SRT

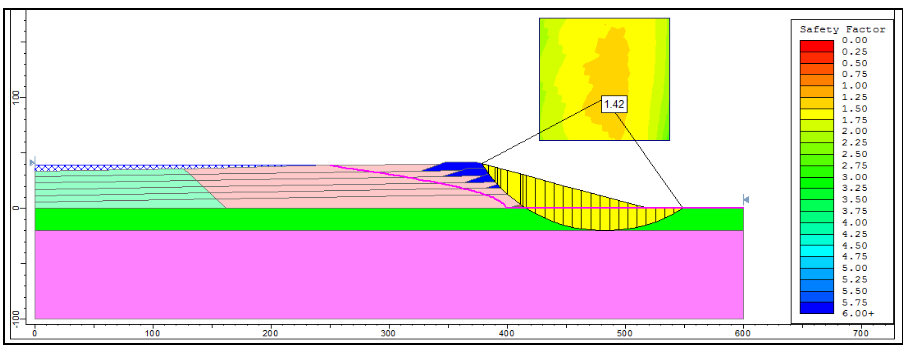
Stage	LEM O-F method		LEM- M-P method		FEM - SRT	
	Static	Pseudo-Static	Static	Pseudo-Static	Static	Pseudo-Static
Starter Dyke	1.30	1.08	1.57	1.28	1.49	1.29
1st Tailings Filling	1.30	1.07	1.57	1.28	1.64	1.59
1st Embankment	1.28	1.06	1.55	1.27	1.63	1.32
2nd Embankment	1.27	1.05	1.50	1.24	1.61	1.28
3rd Embankment	1.25	1.04	1.46	1.21	1.61	1.24
4th Embankment	1.24	1.04	1.44	1.20	1.51	1.23
End of Construction	1.24	1.04	1.42	1.19	1.44	1.20

Table 5 – Static and pseudo-static deterministic FOS for the WRTD using LEMs and SRT

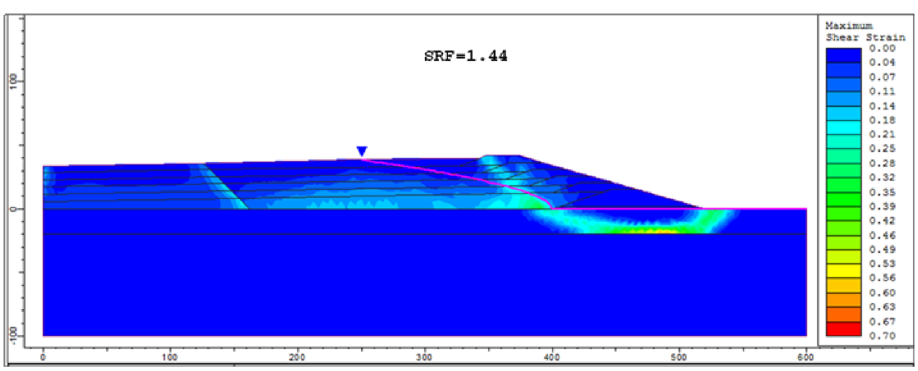
Stage	LEM O-F method		LEM- M-P method		FEM - SRT	
	Static	Pseudo-Static	Static	Pseudo-Static	Static	Pseudo-Static
Full Dam Const.	1.63	1.42	1.70	1.50	1.60	1.40
1st Tailings Fill	1.63	1.41	1.70	1.49	1.62	1.60
2nd Tailings Fill	1.62	1.40	1.70	1.47	1.60	1.58
3rd Tailings Fill	1.58	1.34	1.64	1.38	1.59	1.50
4th Tailings Fill	1.54	1.30	1.58	1.33	1.59	1.48
Dyke	1.54	1.30	1.58	1.32	1.56	1.48
End Construction	1.54	1.30	1.58	1.30	1.54	1.47

The results in Tables 4 and 5 show variations in the FOS in different ways: a) by stage of construction; it is observed that both in the static and pseudo-static states, the FOS decrease with increasing embankment or tailings height. b) The FOS varies within the rigorous and simplified LEMs at a same stage of analysis. For example, the static FOS of the USTF at the end of construction was 1.42 using the Morgenstern Price method, whereas, with the Ordinary-Fellenius method a FOS of 1.24 was obtained.

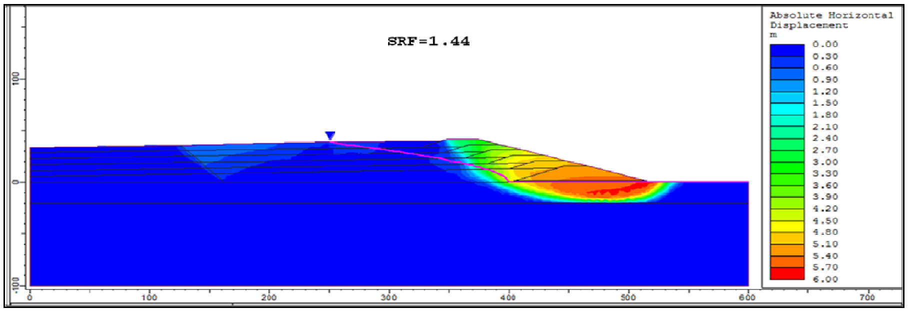
These results demonstrate that TSFs stability analyses deterministic LEM require more than one method to avoid over or underestimating the design FOS. In all cases, however, at least a rigorous LEM needs to be used. Generally, a good correlation between the Morgenstern-Price LEM and the SRT FOS was found but the Morgenstern- Price LEM yields more conservative results. The SRT provides more reliable results because displacement and stress distributions are simultaneously computed with the FOS. The Global minimum slip surface and the contours of maximum shear strain and horizontal displacement of the UTSF for the static analysis are presented in Figure 3a to 3c, respectively.



(a) Global minimum slip surface using Morgenstern-Price LEM



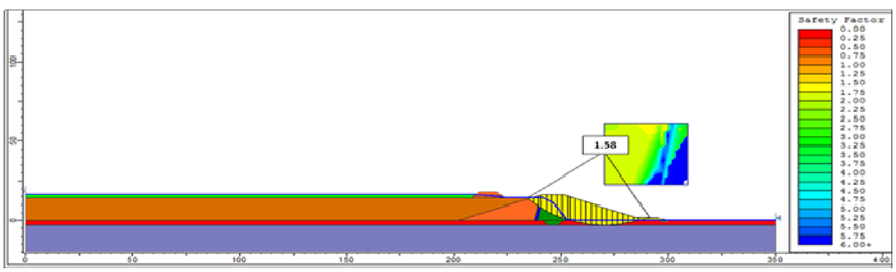
(b) Maximum shear strain using SRT



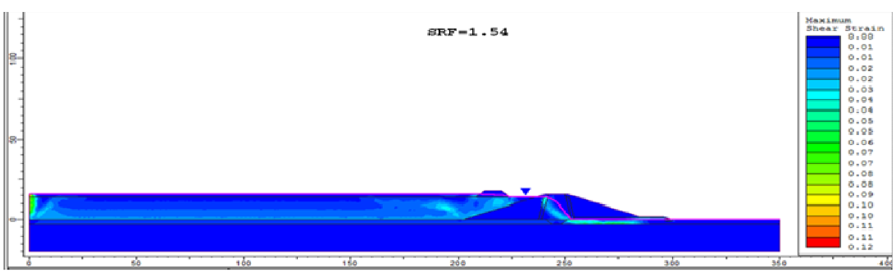
(c) Horizontal displacements using SRT

Figure 3 –SRT stability analysis for UTSF

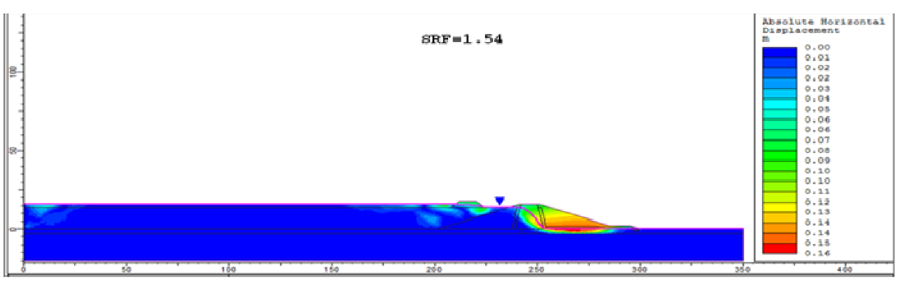
As can be seen in Figures 3a and 3c, there is a good agreement between the location of the global slip surface and the zones of maximum shear strain within the UTSF. Figure 3b shows that the maximum deformation takes place in the top foundation region of the UTSF. Figure 4c shows that the UTSF could reach a horizontal displacement of 6m if sliding occurs. Figure 4a depicts the WRTD global slip surface covers mainly the core and top foundation regions of the WRTD. Figure 4b shows that the maximum deformation is concentrated in these same zones. Figure 4c shows a horizontal displacement of 0.14 m for the WRTD, which is much smaller than the probable displacement of the USTF.



(a) Global minimum FOS –Morgenstern-Price LEM



(b) Maximum shear strain



(c) Total displacements

Figure 4 – SRT stability analysis for WRTD

The difference in displacements between the two TSFs is attributable to the low self-weight consolidation rate of tailings which affects the strength gain processes in UTSFs. Since, the WRTD's embankment is constructed prior to the beginning of operations, materials are allowed to settle and consolidate evenly; this also implies higher strength and isotropic properties for the WRTD. The core, as it is keyed into the bedrock, aids in the overall stability as reflected in the WRTD's strain and displacement figures.

Sensitivity Analyses

The sensitivity analyses helped confirm that the top foundation's friction angle for the UTSFs, and the WRTD core's friction angle and top foundation's cohesion are the parameters governing the overall stability of each type of impoundment. Consequently, these three variables were adopted as the random variables for the MCS and PEM probabilistic stability analyses. The UTSF's top foundation friction angle has a mean value of 21° and a standard deviation of 5.25°. The WRTD's core has a mean value of 28° and a standard deviation of 7°. The top foundation's cohesion of the WRTD has a mean value of 50 kPa and a standard deviation of 12.5 kPa. All random variables were defined as independent and following a normal distribution.

Probabilistic stability analyses using MCS and PEM

The results of the MCS and PEM analysis are summarized in Tables 6–8. The probability density functions of the FOS presented in Figures 5a and 5b show larger areas under the curve in the failure zone for the UTSF under static and pseudo-static states compared to the WRTD. Likewise, the smaller FOS and reliability indices indicate a higher probability of failure and poor to hazardous levels of performance for the UTSF.

Table 6 – Probabilistic analysis–UTSF–1000 MCS- FOS-Morgenstern-Price LEM

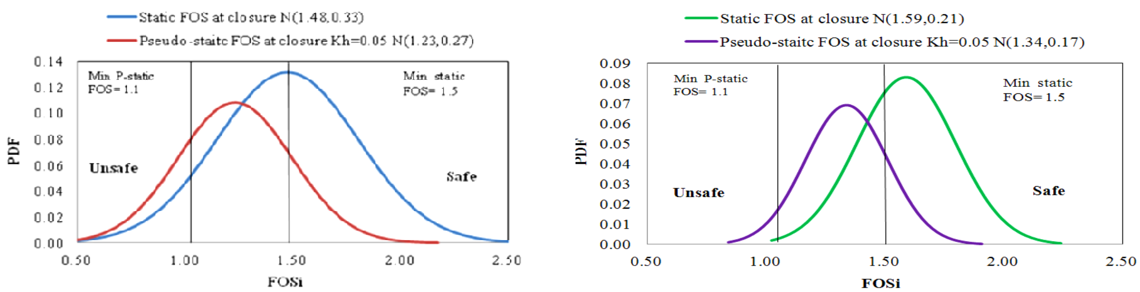
	Static				Pseudo-static			
	Mean FOS	σ	P_f	β	Mean FOS	σ	P_f	β
Starter Dyke	1.61	0.36	0.03	1.71	1.32	0.30	0.14	1.05
1st Tailings Filling	1.60	0.36	0.04	1.66	1.31	0.30	0.14	1.01
1st Embankment	1.59	0.37	0.05	1.61	1.30	0.30	0.15	0.99
2nd Embankment	1.54	0.36	0.06	1.48	1.27	0.29	0.16	0.92
3rd Embankment	1.51	0.36	0.07	1.43	1.24	0.29	0.18	0.84
4th Embankment	1.48	0.34	0.07	1.41	1.23	0.28	0.19	0.82
End Construction	1.48	0.33	0.07	1.40	1.23	0.27	0.22	0.86

Table 7 – Probabilistic analysis–WRTD–1000 MCS- FOS-Morgenstern-Price LEM

Stage	Static				Pseudo-static			
	Mean FOS	σ	P_f	β	Mean FOS	σ	P_f	β
Full Dam Const.	1.78	0.21	0.00	3.71	1.56	0.19	0.000	2.94
1st Tailings Fill	1.78	0.21	0.00	3.62	1.55	0.18	0.000	3.05
2nd Tailings Fill	1.74	0.21	0.00	3.54	1.47	0.17	0.001	2.76
3rd Tailings Fill	1.66	0.20	0.00	3.37	1.40	0.16	0.008	2.50
4th Tailings Fill	1.60	0.19	0.00	3.18	1.34	0.15	0.011	2.26
Dyke	1.59	0.19	0.00	3.11	1.34	0.14	0.011	2.11
End Construction	1.59	0.21	0.00	2.96	1.34	0.17	0.024	1.95

Table 8 – Static and pseudo-static probabilistic analysis –WRTD and UTSF -PEM-SRT K=0.05

Parameter	WRTD		UTSF	
	Static	Pseudo-static	Static	Pseudo-static
Mean FOS-PEM	1.58	1.42	1.42	1.18
Standard Deviation FOS	0.14	0.19	0.38	0.38
P_f	0.00	0.013	0.14	0.32
β	4.30	2.21	1.10	0.47



(a) UTSF **(b) WRTD**

Figure 5 – Probability density functions (PDF) of the FOS-1000 MCS-Morgenstern-Price LEM

CONCLUSIONS

Static and pseudo-static stability analyses of two typical TSF are conducted using the Limit Equilibrium Method and the finite element Strength Reduction Technique, employing the deterministic and probabilistic approaches. Overall, the water retention tailings dam exhibits smaller horizontal displacements, lower shear strain levels, and hence a larger factor of safety, as well as smaller probability of failure, and higher reliability indices compared to those of the upstream TSF for increasing embankment of tailings height. It was found that the factor of safety varies within the rigorous Morgenstern-Price and simplified Ordinary-Fellenius LEMs for identical analytical criteria and stage of analysis. Generally, a better correlation of the factor of safety obtained with the Morgenstern-Price Limit Equilibrium Method and Strength Reduction Technique was observed. However, a higher degree of confidence is placed on SRT results because information about displacement, deformation, stress, and pore pressure distribution are obtained along with the value of FOS. The pseudo- static analysis confirms larger displacements and significant reduction in the FOS at all stages of analysis and methods for both tailings facilities. The Monte Carlo and Point Estimate probabilistic analyses show lower factors of safety, higher probability of failure, and lower reliability indices for the upstream tailings dam. This could be attributed to the intrinsic low consolidation rate of deposited tailings. It is also observed that the overall stability of both types of tailings facilities is governed by the phreatic surface near or on the embankment, the overall pore pressure distribution, and the operative efficiency of the drainage systems.

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