Paper 56

Simulated Annealing For Airborne EM Data Interpretation

Yin, C. [1], Hodges, G. [1]

1. Fugro Airborne Surveys, 2270 Argentia Rd, Mississauga, ON, Canada

ABSTRACT

The traditional algorithms for airborne EM data interpretation, e.g., the Marquardt-Levenburg method, generally take the negative gradient of an objective function as the searching direction and run only downhill search. Such algorithms are strongly dependent on the starting model and can easily be trapped in local minima. Simulated annealing (SA) exploits the analogy between the annealing process of a melted metal cooling and freezing into a crystalline structure of minimum energy, and searches for a minimum in a mathematical optimization. It employs a random search that accepts not only the model updates that decrease the system energy, but also those that increase the system energy. The latter helps the searching process jump out of local minima and converge to a global minimum. During the random walk through the model space in the annealing process, no preferred searching direction is required as for the traditional algorithms. This avoids the calculation of Jacobian derivatives. Further, the SA process allows the starting model to be far away from the true model and thus removes the strong dependency of the results on the starting model. In this paper, we implement the SA for airborne EM inversion, where the starting model is allowed to vary in a large range. We address the SA technique by comparing the airborne EM inversion with a thermodynamic process and discuss specifically the SA procedure from model configuration, random walk for model updates, objective function, and annealing schedule. We examine the effectiveness of the algorithm for airborne EM by inverting both theoretical and survey data and comparing the results with those from the traditional algorithms.

INTRODUCTION

With airborne EM (AEM) systems being widely used in geological mapping for environmental and engineering purposes, more and more attention has been paid to data interpretation and inversion. While the airborne EM inverse problem is non-linear, even for the simplest 1D earth model, a linearization of the problem and an iteration process are generally required for the model solution. The traditional algorithms used in AEM inversion, e.g., the Marquardt-Levenburg method, minimize the objective function defined by the fitting error between the data and the model results. The searching direction of these algorithms for the inverse model is defined by the Jacobean matrix as the negative gradient of the objective function. Thus, the search is always downhill. For appropriate starting models (close to the true model), the search can easily converge to the true solution. However, AEM inversion is non-unique, meaning that a desired global minimum for the true solution is hidden among many local minima. Due to the greedy character of downhill search, the solutions from the traditional algorithms are easily trapped in the local minima.

In this paper, we investigate the simulated annealing (SA) global-searching algorithm for airborne EM inversion by comparing the procedure to a thermodynamic system. The major advantage of SA over traditional algorithms is its ability to avoid becoming trapped in local minima by allowing both downhill and uphill searches. In the past, SA has been successfully used in dc and IP inversion (Sen and Stoffa, 1995; Chunduru et al., 1996). Not much attention has been paid to airborne EM inversion. The simulated annealing for mathematical optimization involves five fundamental aspects: Boltzmann probability, model configuration, random walk for model update, objective function, the temperature and annealing schedule. We discuss these separately by specializing each to airborne EM. We demonstrate the effectiveness of SA for airborne EM inversion by inverting both synthetic data and survey data from Texas Levees area by Fugro RESOLVE system and comparing the results with the traditional techniques.

BOLTZMANN PROBABILITY

The SA scheme for optimization is called the Metropolis procedure (Kirkpatrick et al. 1983). The procedure for airborne EM can be compared to the cooling process of a thermodynamic system. Except for the temperature, the states of the thermodynamic system are analogous to the airborne EM model solutions; the system configuration change is analogous to the AEM model update. While the energy for the system is analogous to the objective function or fitting errors, the ground state in the thermodynamic system is analogous to the global

In "Proceedings of Exploration 07: Fifth Decennial International Conference on Mineral Exploration" edited by B. Milkereit, 2007, p. 843-846

minimum of AEM inversion. In the SA procedure, an initial state of the system is chosen at energy E and temperature T. While holding T constant, the initial configuration is perturbed and the change in energy ΔE is computed. If the change in energy is negative (energy reduced), the new configuration is unconditionally accepted (with probability of 1). If the change in energy is positive (energy increased), it is accepted with a probability given by the Boltzmann distribution

$$p = \exp(-\Delta E / bT), \quad (1)$$

where b is the Boltzmann constant. This process is repeated for sufficient times to obtain good sampling statistics for the current temperature. Then, the temperature is decremented according to an annealing schedule and the entire process repeated until a frozen state is achieved. The Boltzmann probability distribution explains how the SA process prevents a system from being trapped in local minima. In fact, when the search goes uphill, the traditional algorithms will stop going forward. However, the SA algorithm allows the search to continue with a probability. This allows the searching process to jump out of the local minima to reach a global one.

MODEL CONFIGURATION

SA model configuration for AEM inversion involves the description of the model by physical parameters of the earth. In frequency-domain helicopter EM, the transmitter and receiver are generally contained within a Kevlar shell (bird) towed below the helicopter. We use the Fugro RESOLVE system and invert the data for five horizontal coplanar (HCP) coil pairs (Fig.1, Yin and Hodges, 2007). The transmitter-receiver separation is 8m, the frequency ranges from 380Hz to 102 kHz. The bird altitude is typically 30m. The EM field for an HCP coil configuration is

$$H_z = \frac{m}{4\pi}\left\{\frac{3z_+^2 - R_+^2}{R_+^5} - T(z_-)\right\}, \quad (2)$$

where m is the dipole moment of the transmitter, $z_\pm = h_o \pm z$, $R_+ = \sqrt{(r^2 + z_+^{\,2})}$, h_o is the bird altitude and $T(z_-)$ is Hankel's integral, involving the earth parameters and the bird geometry (Yin and Hodges, 2007). In airborne EM, the primary field (the first term in the above equation) is generally stripped off, while the secondary field is normalized by the primary field and expressed as parts per million (ppm) of the primary field, i.e. $ppm = (H_x - H_o) / H_o \times 10^6$.

RANDOM WALK FOR MODEL UPDATE

The configuration change of a system means the update of model parameters to create a new system configuration from the original one. In our airborne EM inversion for an L-layer one-dimensional earth, we have 2L parameters. Since all earth parameters should be positive, there are substantial advantages in using the logarithm of these parameters. This ensures the positiveness of all earth parameters while the inverse model parameters are allowed to change between $-\infty$ and $+\infty$. Thus, for an L-layer earth, the model parameters used in the EM inversion of this paper are $X = (x_1, x_2,.., x_{2L}) = (\ln\rho_1,.., \ln\rho_L, \ln h_1,..,\ln h_{L-1}, \ln h_o)$ with $\rho_1, \rho_2,.., \rho_L$ and $h_1, h_2,.., h_{L-1}$ denoting the resistivities and thicknesses of the earth, while h_o denotes the EM bird altitude. In mathematical optimization using SA, the system configuration change is achieved randomly, meaning that the process takes random walks through the model space, looking for points with low energy. Refer to equation 1, if the system energy is reduced, i.e. $\Delta E \le 0$, then p=1, and the new system configuration is unconditionally accepted (downhill search). If the system energy is increased (uphill search), the probability of taking the step is determined by the Boltzmann distribution. The choice of the random walk for model update is problem specific. Since the SA was introduced into mathematical optimization, a lot of random walk procedures have been suggested. For our AEM inversion, we take the scheme for model perturbation based on the temperature-dependent Cauchy distribution (Sen and Stoffa, 1995), i.e.

$$x_i^{k+1} = x_i^k + y_i(x_i^{\max} - x_i^{\min}), \quad i = 1,2,...2L-1, \quad (3)$$

$$y_i = sign(u_i - 1/2)T_i\,[(1 + 1/T_i)^{|2u_i - 1|} - 1], \quad (4)$$

where i and k denote respectively the ith earth parameter and kth annealing step, while $x_i^{\max}$ and $x_i^{\min}$ denote the upper and lower boundary for ith earth parameter. T_i is the temperature for parameter i, while u_i is a random number drawn from a uniform distribution in the range [0,1].

OBJECTIVE FUNCTION

In simulated annealing, the objective function computes the energy for any given system state. For the airborne EM inverse problem, we define the SA objective function as the fitting error, i.e.

$$E = \sqrt{\frac{1}{2M}\sum_{i=1}^{M}[(\frac{\mathrm{Re}_i - \mathrm{Re}_{0i}}{|H_{z0}|})^2 + (\frac{\mathrm{Im}_i - \mathrm{Im}_{0i}}{|H_{z0}|})^2]}, \quad (5)$$

where M is the number of frequencies, Re_i and Im_i are respectively the theoretical in-phase and quadrature responses for frequency i, while $H_{z0} = (\mathrm{Re}_{0i} + j\,\mathrm{Im}_{0i})$ are the survey data.

ANNEALING SCHEDULE

The annealing schedule for SA process comprises an initial temperature and the rules for lowering it as the search progresses. The temperature and annealing schedule are problem specific. Since SA was introduced into the mathematical optimization, a lot of schedules were suggested for the cooling process. For our purpose of airborne EM inversion, we implement the exponential cooling schedule (Chunduru et al., 1996)

$$T_k = T_0 \exp(-ck^{1/N}) , \qquad (6)$$

where T_0 is the initial temperature, c and N are constants. These parameters may vary for different model parameters, which are generally determined by experiments. Sen and Stoffa (1995) showed that by combining the above annealing schedule with the model update scheme in equations 3 and 4, a global minimum could be statistically obtained. Based on experiments with synthetic data, we choose in this paper c=1, N=2 for all model parameters, while T varies for different parameters. As it will be seen, the cooling schedule in equation 6 combining model updates in equations 3 and 4 yields stable and fast solutions for both synthetic and survey data.

MODELRESULTS

Synthetic data

To test the SA algorithm introduced above and to obtain optimal annealing parameters for airborne EM inversion, we assume a two-layer earth model. The top layer has a resistivity of 10 ohm-m and a thickness of 10m. The underlying half-space has a resistivity 100 ohm-m. The bird altitude is 30m. The EM responses for 5 HCP frequencies (380, 1400, 6200, 25000, 102000Hz) of the Fugro RESOLVE system are calculated and used as data for the inversion. For the starting model, all earth parameters are allowed to vary in a large range. From Figure 1, one sees that, for the assumed range of starting models, all parameters are resolved after 250 iterations (lowering initial temperature). Specifically, for the range of 1-20 ohm-m for ρ_1, 10-200 ohm-m for ρ_2, and 0-60m for h_1, all searches converge to the true value. The fitting error reduces continuously from over 50% initially to below 0.2%. The good results show that the right choice on the SA schedule and temperature has been made for airborne EM data.

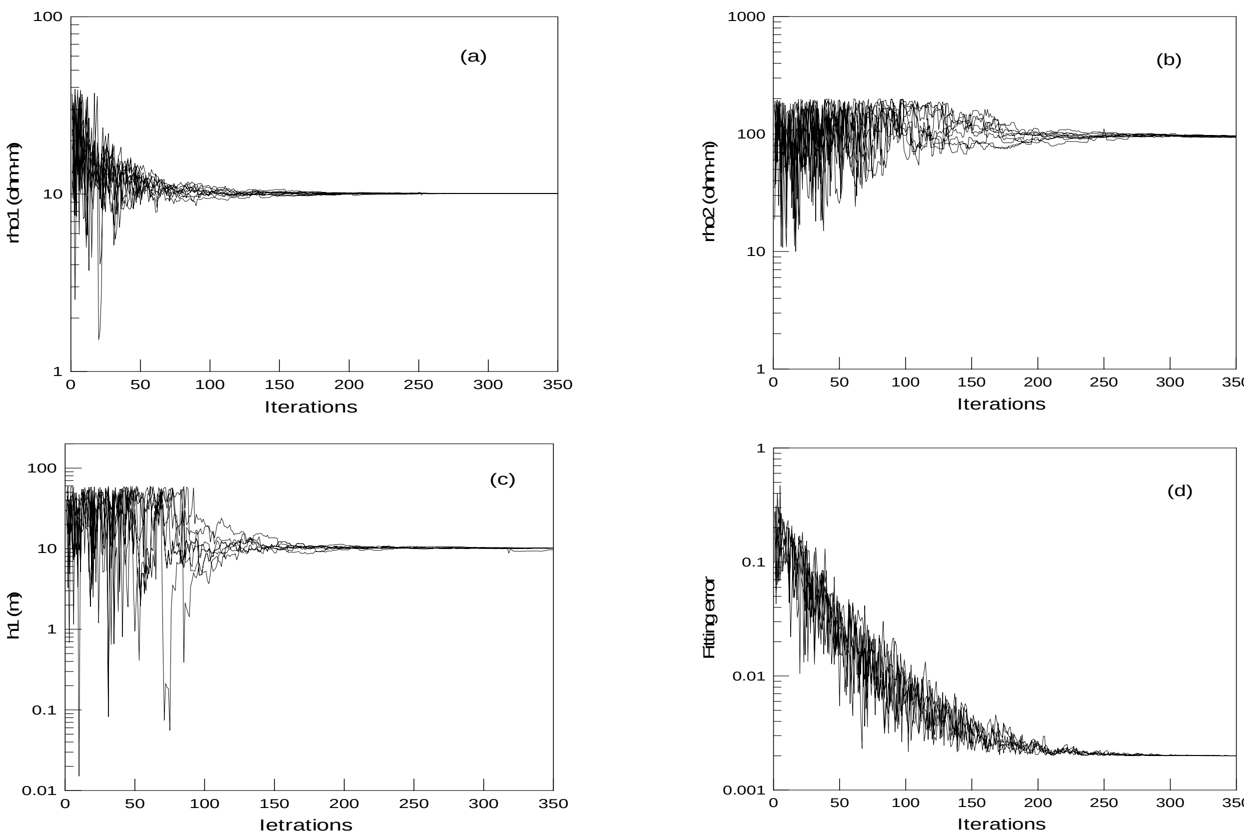

Figure 1: Inversion results by simulated annealing for a theoretical two-layer model. (a) First layer resistivity; (b) second layer resistivity; (c) first layer thickness; (d) fitting error.

Field data

Figure 2 shows the inversion results of a survey by Fugro RESOLVE system over Rio Grande Levees near Brownsville, Texas for the U.S. Army Corps of Engineers. Each inversion was run with 20 different starting models within the range of 5 times greater or less than a best estimate of the resistivity and thickness for each layer. The average time for 20 runs (each with 400 temperatures, 40 random walks per temperature) was 5 minutes. For comparison, the sections obtained by CDT (conductivity-depth-transform) and SVD inversion (Hodges, 2003) are also displayed. It is seen that the simulated annealing algorithm gives a more continuous and stable result than the other two algorithms.

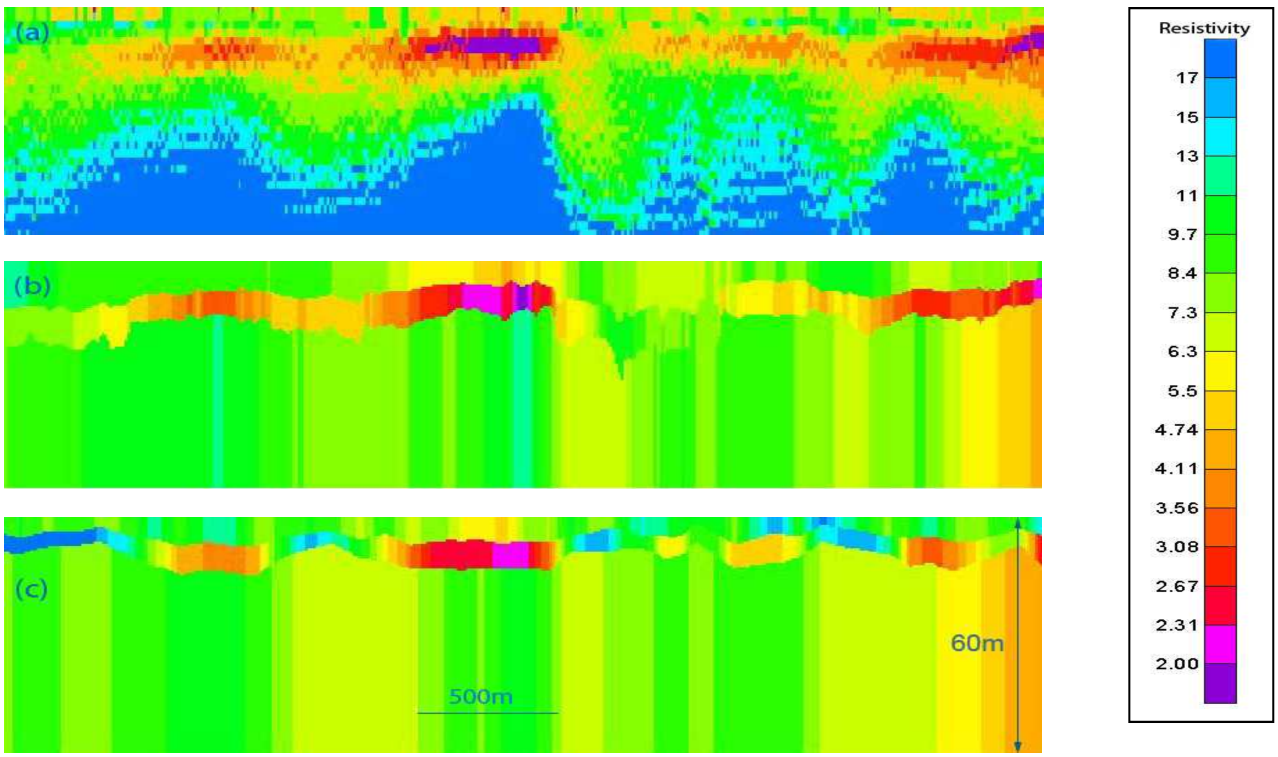

Figure 2: Inversion results from the survey data in Texas Levees area. (a) CDT section; (b) 3-layer SVD inversion; (c) 3-layer SA inversion. The resistivity unit is ohm-m. Data courtesy of U.S. Army Corps of Engineers.

CONCLUSIONS

In comparison to traditional inversions, the simulated annealing technique offers more stable results due to its global search scheme. The starting model for SA can be selected as a range, rather than the traditional assumption of single value for each parameter. Thus, the dependence on accurate starting models is reduced. Both the theoretical and survey data inversions show that the designed SA procedure works well for AEM data. However, the annealing schedule, initial temperature, and model update are all model specific. Furthermore, in comparison to the traditional local searching algorithm, the model calculation for SA can be heavy. This means that a fast modeling algorithm is required to ensure the quick search.

REFERENCES

Chunduru, R.K., Sen, M.K., and Stoffa, P.L. 1996, Resistivity inversionfor 2-D geologic structures using very fast simulated annealing:Geophysics, 61, 151-161.

Hodges, G., 2003, Practical inversions for helicopter electromagnetic data: SAGEEP Proceedings, 45-58.

Kirkpatrick, S., Gelatt, C.D., and Vecchi, M.P., 1983, Optimization by simulated annealing: Science, 220, 671-680.

Sen, M.K., and Stoffa, P.L., 1995, Global optimization methods in geophysical inversion: Elsevier, Science Publication Co.

Yin, C., and Hodges, G., 2007, 3D animated visualization of EMdiffusion for a frequency-domain helicopter EM system:Geophysics, 72, F1-F7.