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RECENT DEVELOPMENTS IN THE USE AND INTERPRETATION OF DIRECT-CURRENT RESISTIVITY SURVEYS

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Abstract

The direct-current resistivity survey method technique has developed slowly since the early part of this century, with applications being primarily for engineering problems and in the search for groundwater. Over the past two decades, extensive resistivity surveys have been carried out in mining exploration, but mainly as an integral part of induced polarization surveys. In the last few years, use of direct-current resistivity surveys has expanded rapidly because of their value in geothermal exploration. In part, geothermal surveys are carried out using conventional Schlumberger and dipole-dipole techniques, but in part, geothermal exploration is being done using arrays which previously had not been greatly utilized. In particular, the dipole mapping array (or, as it is also known, the bipole-dipole array), has been used extensively to map two- and three-dimensional resistivity structures. This expanded use of the DC resistivity method has led to increased concern with interpretation procedures. Numerical methods for the interpretation of layered structures are well developed, but the application of such methods to two- and three-dimensional structures is still at an early stage.

Résumé

La technologie des levés géophysiques par les méthodes de résistivité en courant continu s'est développée peu à peu à partir du début de ce siècle; les premières applications concernaient surtout les problèmes liés à l'ingéniérie et à la recherche des eaux souterraines. Au cours des deux dernières décennies, des levés de résistivité assez étendus ont été exécutés dans le cadre de campagnes de prospection minière, mais ces levés faisaient le plus souvent partie intégrante de levés de polarisation provoquée. Ces dernières années, l'emploi des levés de résistivité en courant continu s'est répandu rapidement, en raison des possibilités qu'ils offrent pour la prospection des ressources géothermiques. Les levés géothermiques se font en partie à l'aide des techniques classiques comme la technique Schlumberger et celle du dipôle-dipôle, mais en partie aussi en utilisant des dispositifs qu'on n'avait que peu utilisés auparavant. En particulier, le dispositif dipôle de levé de surface aussi connu sous le non de bipôle-dipôle a été largement utilisé pour établir la carte ou le bloc-diagramme de certaines structures de résistivité. Du fait de ce regain de la méthode de résistivité en courant continu, il a fallu apporter encore plus d'attention aux méthodes d'interpértation. Les méthodes numériques d'interprétation des structures stratifiées sont parfaitement au point, mais l'application de ces méthodes à des structures représentées en deux ou en trois dimensions n'en est encore qu'à ses débuts.

INTRODUCTION

Traditionally, electrical prospecting methods based on the use of direct current have been considered separately from those based on the use of alternating currents. Often, the term "resistivity method" is used to indicate a method based on the assumption of direct current flow in the earth. The use of this definition is no longer satisfactory inasmuch as earth resistivity is now being measured using alternating currents as well as direct currents. In this paper, I will discuss those methods for measuring earth resistivity in which (the assumption is made that) direct current is used to energize the earth. In actual fact a low-frequency alternating-current is most frequently used in field practice. The assumption of direct current merely requires that the frequency be low enough that magnetic coupling between current flow lines can be neglected, and the flow of current in the earth can be described adequately by Laplace's equation.

Roman (1960) cited the earliest uses of electrical resistivity methods as being due to Fox, prior to 1830, and to Barus, who in 1883 reported on the use of electrical methods in mapping extensions of the Comstock Lode in Nevada. Electrical resistivity methods developed slowly from these early beginnings and as Roman pointed out, although the results have never been spectacular, successful resistivity measurements have been made by many people for many purposes.

The induced polarization method (IP) is the most popular and the most effective of the electrical exploration methods used in the search for metallic ore deposits. It is impractical in most cases to do an IP survey without simultaneously measuring earth resistivity. Often, the earth resistivity data so obtained are useful in characterizing the nature of a rock mass which gives rise to an induced polarization effect. However, for this paper, the application of resistivity methods in parallel with induced polarization will not be considered.

In the last decade, the independent use of electrical resistivity methods has grown very rapidly in comparison to the extent of use in previous decades. Principal applications have been in the solution of engineering problems and in the search for groundwater. However, perhaps the greatest growth has occurred since electrical resistivity surveys have come into use in exploration for geothermal resources. Here, the methods are the primary approach to locating geothermal resources. Because of this, significant changes have been taking place both in the way in which resistivity surveys are carried out in the field and the way in which the data are interpreted to yield information about geologic structure.

Two trends can be recognized in the increased application of electrical resistivity methods. First, a wider variety of field techniques is being tried, and some of these new techniques are becoming adopted in general use. Second, significant changes in the way in which resistivity surveys are used have come about as a consequence of theoretical developments. Computational methods and analytical procedures are now available which permit a far more rigorous interpretation of field observations than has been possible in the past. Interpretation of resistivity data is less subjective and interpretations can now be made with greater detail than was previously possible. An important added feature of some of the new data processing methods is that the reliability of the interpretations can be estimated.

In the paper which follows, the development of new field techniques, and particularly the application of new interpretive techniques, will be reviewed.

FIELD TECHNIQUES

Most direct current methods for measuring resistivity employ an array of four electrodes in contact with the ground. Two electrodes are used to provide current to the ground, while the other two are used to measure the voltage developed by this current flow in the ground. The geometry of electrode layout is used to control the sensing area and depth of investigation of the resistivity measurement. While it is quite possible to compute an apparent resistivity value for measurements made with an array of completely arbitrarily located electrodes, in practice, a few standardized arrays are used much more commonly than others. These include the Wenner array, the Schlumberger array, the dipole-dipole array, and more recently, the general dipole array. All but the last have been thoroughly described in textbooks on resistivity surveying, including Keller and Frischknecht (1966), Orellano (1972), Dobrin (1976), and Telford et al. (1976).

The general dipole or bipole-dipole method was first described in detail by Alpin (1966), who described the use of a dipole current source and a dipole receiver arbitrarily positioned with respect to each other as a means for measuring earth resistivity. The advantages for the dipole method were operational ease in that the lengths of cable that had to be laid out were relatively short, and a relatively great sensitivity to lateral changes in resistivity in comparison with that of other electrode arrays.

Alpin employed a short separation between the current electrodes so that the field would behave as though it originated from a dipole source of zero length. In the subsequent development of bipole-dipole methods, it has become more common to use a source whose length is too great to permit the dipolar approximation. One of the first bipole-dipole surveys is described by Stefanescu and Tanasescu (1965) and by Doicin et al. (1965). In a survey of potentially oil-bearing strata in Transylvania, a bipole source which was actually an out-of-service power line several tens of kilometres in length was used to energize the area under study. The electric field distribution about this bipole source was then mapped over an area of nearly 1000 km^2. The results were said by the authors to provide an excellent representation of the structure of the sedimentary basin within the area surveyed.

In 1967 and 1968, a similar bipole-dipole resistivity survey was carried out in the vicinity of the Broadlands geothermal prospect in the central part of the north island of New Zealand (Risk et al., 1970). Two bipole current sources oriented more or less at right angles to one another were used sequentially to energize the area of the survey. The electric field was mapped in detail over an area of approximately 50 km^2. The survey outlined a region of low resistivity which is closely associated with a geothermal reservoir at depths of one kilometre and more.

Furgerson (1970) reports the use of a bipole-dipole resistivity survey in the Darwin Hills area in southeast California to map the geometry of a small intrusive associated with the Darwin Hills ore deposits. Again, the method appeared to be effective in locating the boundaries between major lithologic units over an area of several tens of square kilometres.

The essential features of the bipole-dipole electrode array are shown in Figure 6.1. The ground is energized by passing current through a relatively long wire grounded at points "A" and "B". The length of the source may range from a kilometre to several tens of kilometres. The amount of current provided to the source must be great enough to produce easily detectable signals over the area to be mapped. The current waveform is usually that of a square wave or an asymmetric square wave with low frequency content so that direct current behavior can be assumed. The amplitude of these square waves may range from a few amperes in a small scale survey to 500 amperes or more in a large scale survey. In order to obtain the required high current levels, it is necessary to expend considerable effort to obtain low resistance ground contacts, particularly in areas where the ground resistivity is high. Existing metal structures, such as well casings and highway culverts, may sometimes be used. In other cases, it may be necessary to drill holes in which to install electrodes, or dig ditches to the water table.

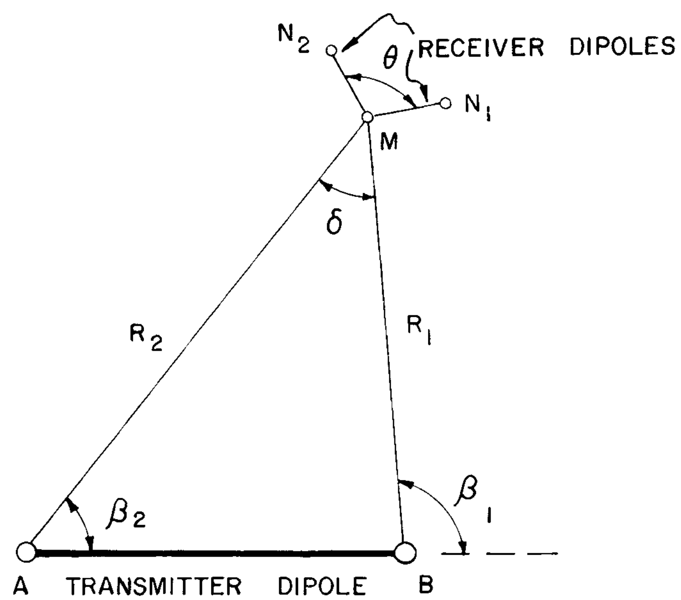


Figure 6.1. *Definition of electrode layout used in bipole-dipole surveys (Arestad, J., 1977).*

A heavy generator with a capacity ranging from ten kilowatts to several hundred kilowatts is required to provide the large currents mentioned above. Such equipment is heavy, and the current source cannot readily be located away from roads.

The electric field generated by this current is mapped over the area of interest. Usually, the electric field is detected by laying out two short receiver dipoles more or less at right angles to measure two components of electrical field. Measurements are made at many locations with the number ranging from 50 to 200 over the area of interest using a single bipole source for excitation. Measured electric field values are converted to apparent resistivity values as is done with other electrode arrays. When this is done, one recognizes that the field technique provides redundant data; that is, more quantities are measured than are necessary to compute a single value of apparent resistivity at each observation point. Sufficient data are recorded in the field to permit

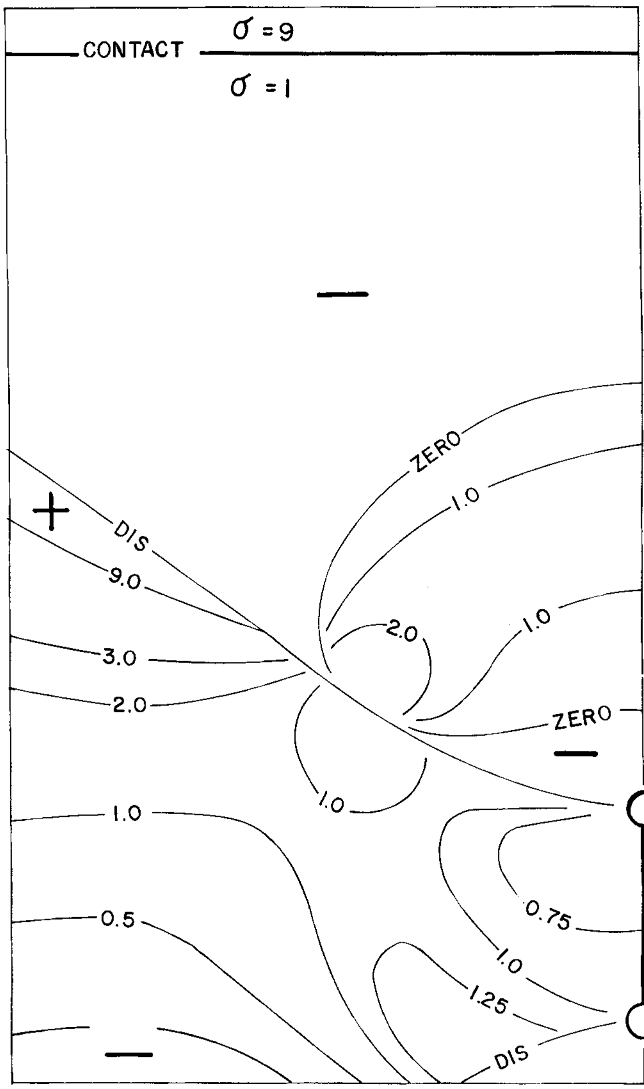


Figure 6.2. Map of apparent resistivity computed for the component of electric field perpendicular to the boundary, in a medium with a single vertical boundary separating regions with unit resistivity and with a resistivity of nine units. The line marked "DIS" represents the locus of points where the component of electric field parallel to the source is zero, and a discontinuity exists in the expression for apparent resistivity (from Furgerson, 1970). d = 4c; k = 0.8.

computation of two independent values of apparent resistivity. For example, Alpin (1966) has given expressions for computing apparent resistivity values for the components of the electric field parallel to and perpendicular to the direction of the bipole source.

Furgerson (1970) computed both parallel and perpendicular apparent resistivity values for his survey of the Darwin Hills. These two values of apparent resistivity have the unusual feature that negative values of resistivity can appear in the data. These are usually characteristic of particular types of lateral change in the actual resistivity in the earth. They may be useful in interpretation, but the theoretical results presented by Furgerson indicate that the complexity of the patterns is so great that interpretation would be difficult. An example of the behavior of parallel and perpendicular resistivities for a simple two-layer structure is shown in Figure 6.2.

Another way of making use of the redundant field observations is to compute an apparent resistivity value from the magnitude of the electric field vector, and consider the direction of the electric field vector as an independent piece of data. At present, this is the mode of presentation most commonly used in presenting bipole-dipole surveys. The expression for computing total field apparent resistivity, as it is called, is as follows

$$\rho_a = \frac{2\pi R_1^2}{\left[1+\left(\frac{R_1}{R_2}\right)^4-2\left(\frac{R_1}{R_2}\right)^2\cos\delta\right]^{1/2}} \frac{|E_T|}{I} \qquad \ldots 1$$

where R_1 and R_2 are the distance from an observation point to the two ends of the bipole source, δ is the angle between these two lines, E_T is the magnitude of the electric field vector at the observation point, and I is the amplitude of the current pulse provided to the ground. Separately, the direction of the electric field vectors can be presented as shown in Figure 6.3. These vectors will be deviated from their normal direction in a manner which will be indicative of the locations of resistivity changes away from the locations where measurements are actually made.

Still another way of using the redundant field data has been suggested by Zohdy (1970). Zohdy suggests treating the bipole source as though it were two independent single pole sources of current. Each pole of the source will contribute one electric field vector at a receiver station. The observed electric field would be the sum of these two vectors. Unless measurements are made along the equatorial axis of the bipole source, the field data are adequate to compute two separate values of apparent resistivity valid for the pole-dipole array, which is the same as the Schlumberger array.

Still another way of presenting bipole-dipole results is in the form of apparent conductance. The definition of apparent resistivity is based on the assumption that the earth is completely uniform. Another model which is sometimes more realistic is one in which the earth consists of a thin layer with a given conductance (the ratio of thickness to resistivity) covering an insulating substratum. In such a case, the value of apparent resistivity increases linearly with separation between bipole source and dipole receiver. This consistent increase can mask the changes in apparent resistivity caused by lateral effects. The lateral effects can be accentuated in these cases by computing an apparent conductance using the formula

$$S_a = \frac{2\pi R_1}{\left[1+\left(\frac{R_1}{R_2}\right)^2-2\left(\frac{R_1}{R_2}\right)\cos\delta\right]^{1/2}} \frac{|E_T|}{I} \qquad \ldots 2$$

where the parameters are the same as defined earlier.

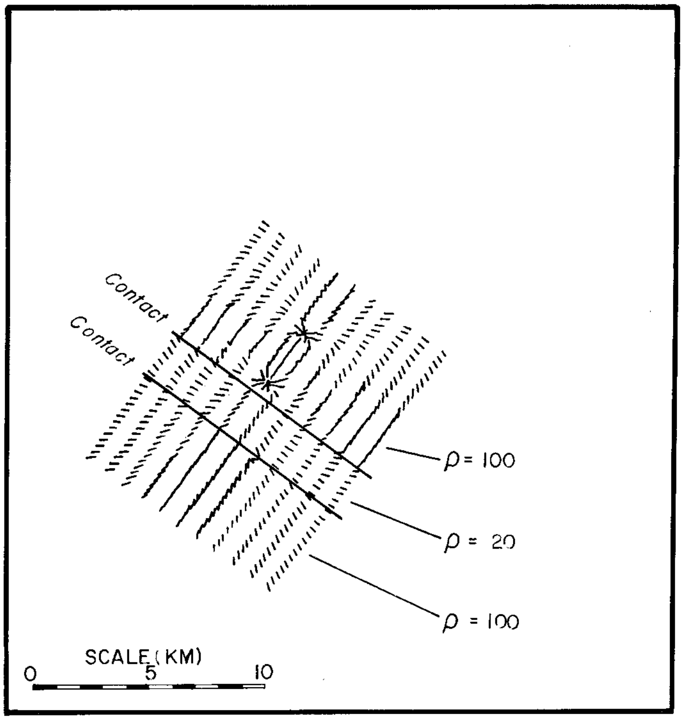


Figure 6.3. *Computed directions for electric field vectors for the case of a vertical conductive slab (Arestad, 1977).*

Both field studies and theoretical studies (Keller et al., 1975) indicate that under some circumstances, the dipole mapping method or bipole-dipole method provides excellent definition of the boundaries of zones with unique resistivity contrasts. In other cases, the method appears to provide confusing or even misleading results (Beyer, 1977). The best results appear to be obtained when the bipole source is located outside the target area when the target has relatively low resistivity compared to the rest of the terrane, or when only one terminal of the bipole source is located within a zone of low resistivity. Poor results are obtained if both ends of the bipole source are located within a relatively conductive feature. In order to avoid such problems, either the general nature of the resistivity structure must be known before the bipole source is located, or several surveys must be carried out with overlapping coverage to assure that at least one bipole source is partially located outside of the conductive features being mapped.

An extension of the bipole-dipole method which provides even more highly redundant data is the crossed dipole method. This was first suggested by Vedrintsev (1966), who proposed using the differences in character of the equatorial and polar-dipole sounding surveys. In field practice, two bipole sources oriented more or less at right angles to one another are established. Then, at a receiver station, two sets of electric field measurements are made, one for excitation of each of the two dipole sources. The results can be treated as though two separate bipole-dipole surveys were being carried out, but much of the advantage of this method lies in the comparison ofthe two electric fields. Morris (1975) described the application of the crossed-dipole method to the mapping of structure in a geothermal prospect in the Black Rock Desert area of northwestern Nevada. In the compilation of the data, he not only computed the apparent resistivities for the two separate sources, but combined the electric field vectors from the two sources in varying proportions so as to simulate apparent resistivity values for current flow in all possible directions at a given receiver station. An example of his field data along with a theoretical model which was used to interpret the field measurements is shown in Figure 6.4A, B.

According to Morris, when resistivity values are computed as a function of the direction of the electrical field vector at a receiver station, an elliptical pattern is generated which he called an ellipse of anisotropy. It is characterized by a maximum and minimum value of apparent resistivity, as well as a direction in which the maximum value of apparent resistivity is determined. He defined a coefficient of anisotropy as being the square root of the ratio of maximum to minimum apparent resistivity. There parameters are defined in Figure 6.5. According to simple theoretical models developed by Morris, plots of the direction of the resistivity ellipse axis, of the coefficient of anisotropy, and of the average of the maximum and minimum apparent resistivities all provide relatively simpler patterns than do the total field resistivities obtained from single dipole coverage.

Doicin (1976) has also carried out a theoretical evaluation of measurements made with two dipole sources oriented at right angles. According to Doicin, the most useful parameter to be obtained from the field measurements is the vector cross product of the two electric fields that are measured. This parameter behaves in much the same way as the average resistivity parameter defined by Morris. A tensor apparent resistivity has been defined by Bibby (1977).

With bipole-dipole mapping, the apparent resistivity computed for a given receiver station usually varies widely for different source locations. When overlapping coverage is provided from several sources to assure that proper excitation is obtained for any conductive features in the area being surveyed, a problem arises when one attempts to present the results on a common map. Multiple values measured at each receiver station will not agree. One solution is to consider each map separately, and obtain compatible interpretations in areas where coverage from several sources overlap. Another approach is to provide sufficiently redundant coverage that averaging of values computed from various sources is possible.

An example of a survey in which averaging appears to work well is shown in Figure 6.6B. Here, the results of a bipole-dipole survey of the Imperial Valley in southern California are shown. Fifty bipole sources were used, placed in pairs at intervals of approximately 10 km, as shown in Figure 6.6A. Approximately 5000 individual values of apparent resistivity were determined over the area of survey. To compile Figure 6.6B, a grid with a spacing of 1.6 km was placed over the surveyed area, and the geometric means for all apparent resistivity values measured within a radius of 1.6 km about each node on the grid were contoured. The presentation provides a coherent picture of major resistivity changes in the Imperial Valley. Arestad (1977) also gives an example of spatial averaging to present the results of many overlapping bipole-dipole surveys.

A technique which is closely related to the bipole-dipole method is the mise-à-la-masse method. In this, a conductive mass of rock is energized by placing one current electrode within it, with the current return being placed at a considerable distance. The electric field behavior is mapped to detect a sudden increase in field strength as current leaves the conductive mass to enter more resistant rock around the outside. The significant difference between the mise-à-la-masse method and the bipole-dipole method as described here is that in the mise-à-la-masse method, commonly the electrode placed in the conductive rock mass is situated in a borehole. An example of the use of the mise-à-la-masse method in recent years is reported by Ketola (1972). The problem of field behavior with the mise-à-la-masse method has been considered in some detail by Merkel (1971), by Snyder and Merkel (1973), and by Merkel and Alexander (1971). They have considered both the case in which the electrode is placed within the conductive mass, and the case in which the electrode is placed close to the outer boundary of the conductive mass in a borehole. In either case, considerable advantage is gained in being able to map the boundaries of the conductive zone.

In summary, the recent increase in field utilization of resistivity methods particularly for exploration in geothermal areas has led to the extensive use of electrode arrays in which electric field behavior is mapped over a planar surface, rather than at a few specified points. This greater amount of information about the structure of the electric field from a bipole source may ultimately lead to the better definition of the electrical structure of the earth in areas where surveys are being carried out.

THEORETICAL DEVELOPMENTS

One-Dimensional Forward Problem

By convention, the computation of the behavior of electric potential when the resistivity structure has been specified is called the forward problem. The inverse of this problem – determining the resistivity structure from observations of the potential field behavior – is the one of interest in interpretation. Sor far, it has been necessary to be able to solve the appropriate forward problem before the inverse interpretation can be carried out. Forward problems are of varying degrees of complexity, ranging from the relatively simple case of a one-dimensional variation of resistivity to the highly complex case of a three-dimensional variation. The one-dimensional forward problem has been studied for a very long period of time, and is still the object of many theoretical analyses. The most common one-dimensional problem considered in earth resistivity studies is the one in which resistivity varies only with depth in the earth. However, a problem in which the resistivity varies only with one of the horizontal directions is equally amenable to solution.

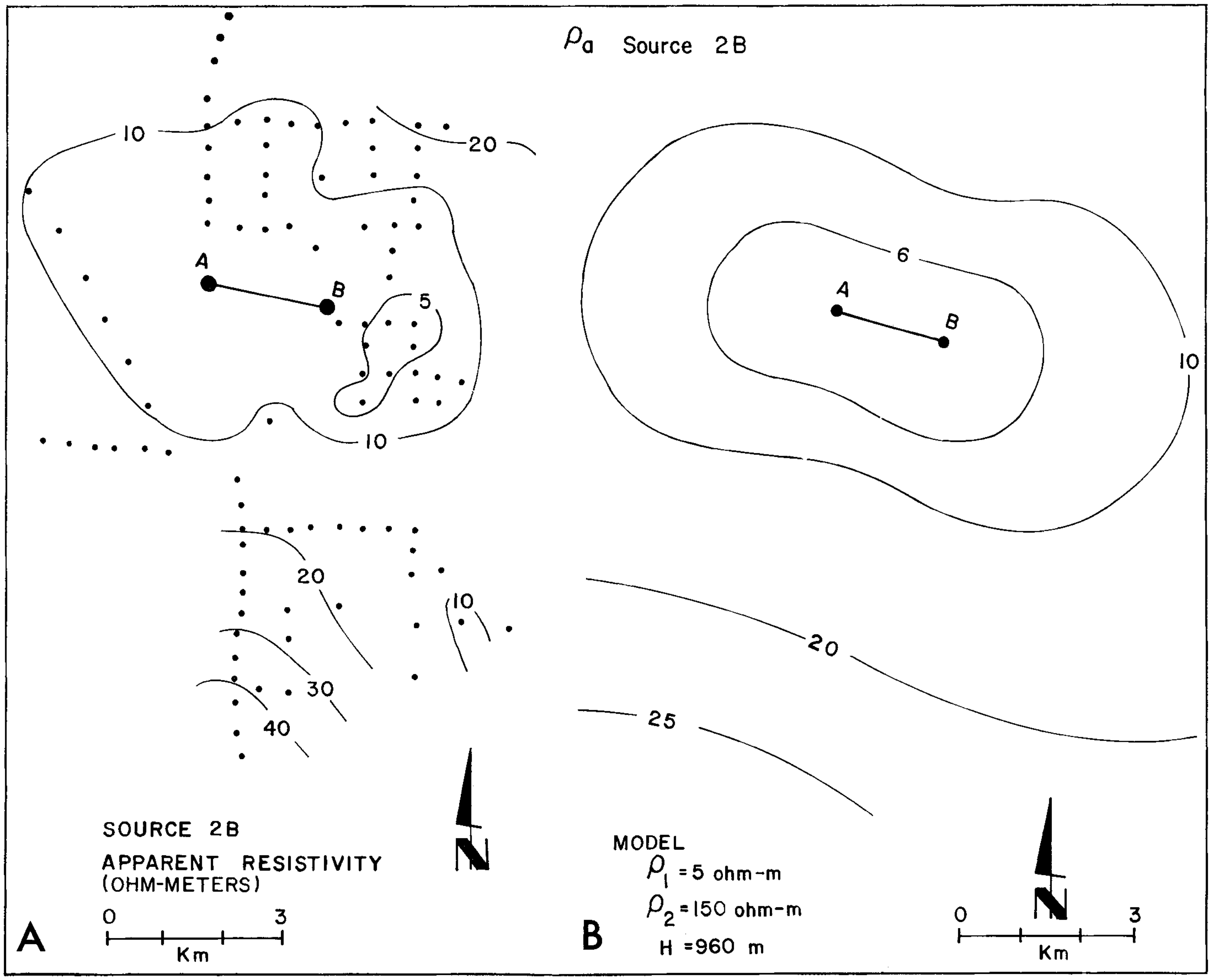
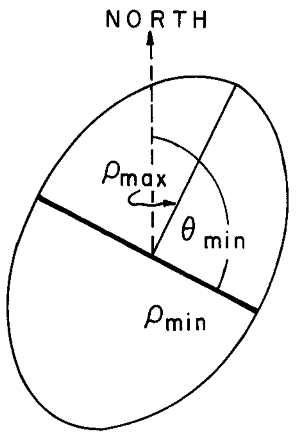


Figure 6.4A. *Apparent resistivity map (total field) for a survey done in northwestern Nevada. Values in ohm-m.*

Figure 6.4B. *Apparent resistivity map computed for a two-layer model to match the data in Figure 6.4A (from Morris, 1975).*



ELLIPTICITY $= \sqrt{\dfrac{\rho_{max}}{\rho_{min}}}$

AVERAGE RESISTIVITY $\rho_{ave} = \dfrac{(\rho_{max} + \rho_{min})}{2}$

BEARING OF MINOR AXIS, θ_{min}, MEASURED CLOCKWISE

Figure 6.5. *Definition of ellipse of resistivity measured with the quadripole resistivity method (Morris, 1975).*

The standard solution for the one-dimensional variation of resistivity with depth which is in use today was apparently first published by Stefanescu et al. (1930). It is based on a solution of Laplace's equation in cylindrical coordinates which leads to an expression for the potential on the surface of the earth in the following form:

$$U = \frac{\rho_1 I}{2\pi} \quad \frac{1}{r} + 2\int_0^\infty \Theta(\lambda)\, J_0(\lambda r) d\lambda \qquad \ldots 3$$

where U is the potential, r is the distance from a single pole source of current to a single measurement point, ρ^1 is the resistivity at the surface of the earth, λ is a dummy variable of integration which enters in the solution of the differential equation, J_0 is a Bessel function of the first kind of order 0, and Θ is a kernel function (Slichter, 1933) which depends on a function of the resistivity structure of the earth. According to Slichter, kernel functions can readily be found for a variety of resistivity distribution functions, including those which are piecewise uniform (a series of layers with fixed resistivities), or a variety of functional relationships between resistivity and depth including an exponential function of depth, a power function of depth, a hyperbolic function of depth, or a trigonometric function of depth. The kernel functions which correspond to each of these variations have been tabulated by Meinardus (1967).

This expression has been evaluated numerous times for the piecewise continuous variation of resistivity with depth that characterizes a sequence of uniform layers. Numerical results have also been obtained for more complex changes in resistivity with depth, including the case of a linear transitional layer in which the resistivity varies linearly from one level to another as reported by Jain (1972), a power law variation within a single layer (Niwas and Upadhyay, 1974), and a layer with an exponential variation of resistivity with depth as reported by Stoyer and Wait (1977). Naidu (1970) has developed an expression for the potential over an earth in which resistivity is a random function of depth characterized by a standard deviation and an autocorrelation function. Lee (1977) has extended this analysis, presented some numerical results, and considered the case of a sequence of random layers resting upon a deterministic lower medium.

The form of the forward solution for the one-dimensional problem as given in equation 3 prevented any extensive numerical computations until high-speed computers became available. The Bessel function within the integral is oscillatory, so that numerical methods of evaluating this integral could not be used prior to the development of high-speed computers. The only cases for which the integral could be evaluated were those in which the integral could be expressed in a closed form (Koefoed, 1966, 1968). The kernel function can be expanded in a Taylor series, with each term being in a form which can be evaluated in a closed form. However, this can be done only when the layers have integer thicknesses and uniform resistivities (Mooney et al., 1966). Expansion of the integral in terms of a series for the kernel function leads to a simple series which has been called a series of images by Roman (1959, 1960, 1963). Even with the image approach for evaluating the integral, computations were tedious so that with hand calculations solutions could be obtained only for a few layers.

With the availability of high-speed computers, it became possible to carry out direct numerical evaluations of the integral in equation 3. Meinardus (1967, 1970) describes a method based on numerical quadrature for evaluating the integral for an arbitrarily large number of layers. While feasible, the method is expensive in terms of computer time. Later, Crous (1971) used a method in which the kernel function is approximated by a spline interpolation formula and the integral in equation 3 is broken into a series of definite integrals which could be evaluated with less effort.

The method which is most commonly used today in evaluating the Hankel transform integral in equation 3 is the convolution method first described by Strakhov (1968) and by Ghosh (1971a, b) and subsequently expanded in papers by Anderson, 1973; Koefoed, 1976a, b, c; Verma and Koefoed, 1973; Das and Ghosh, 1973, 1974; and Das et al., 1974. In this approach the Hankel Transform Integral is converted to the form of a convolution integral by making the substitutions

$$x = \ln\lambda$$
$$y = \ln\left(\frac{1}{r}\right) \qquad \ldots 4$$

in equation 3 to yield the result

$$U = \frac{\rho_1 I}{2\pi}\left[y + 2\int_0^\infty T(x) e^x J_0(e^{x-y}) dx\right] \qquad \ldots 5$$

where T(x) is the kernel function expressed in terms of the transformed variable, x. This integral has the form of a convolution, which is a rapid numerical procedure on a high-speed computer, and efficient programs making use of linear filtering are available (Anderson, 1973). The filter representing the Bessel function is determined by considering the Hankel transform of any analytical kernel for which an analytical answer is available, such as $e^{-a\lambda}$ and determining the filter numerically by division of the Fourier transforms. The exact nature of the filter operator will depend on the particular combination of Bessel functions in the Hankel transform, on the sampling density, and on the length of the operator. Sampling densities ranging from three points per decade of λ to ten points per decade have been used (Anderson, 1973; Daniels, 1974). The length of the filter operator may vary from nine points (Strakhov, 1968) to forty points (Anderson, 1973) or longer. The convolution approach

Contents

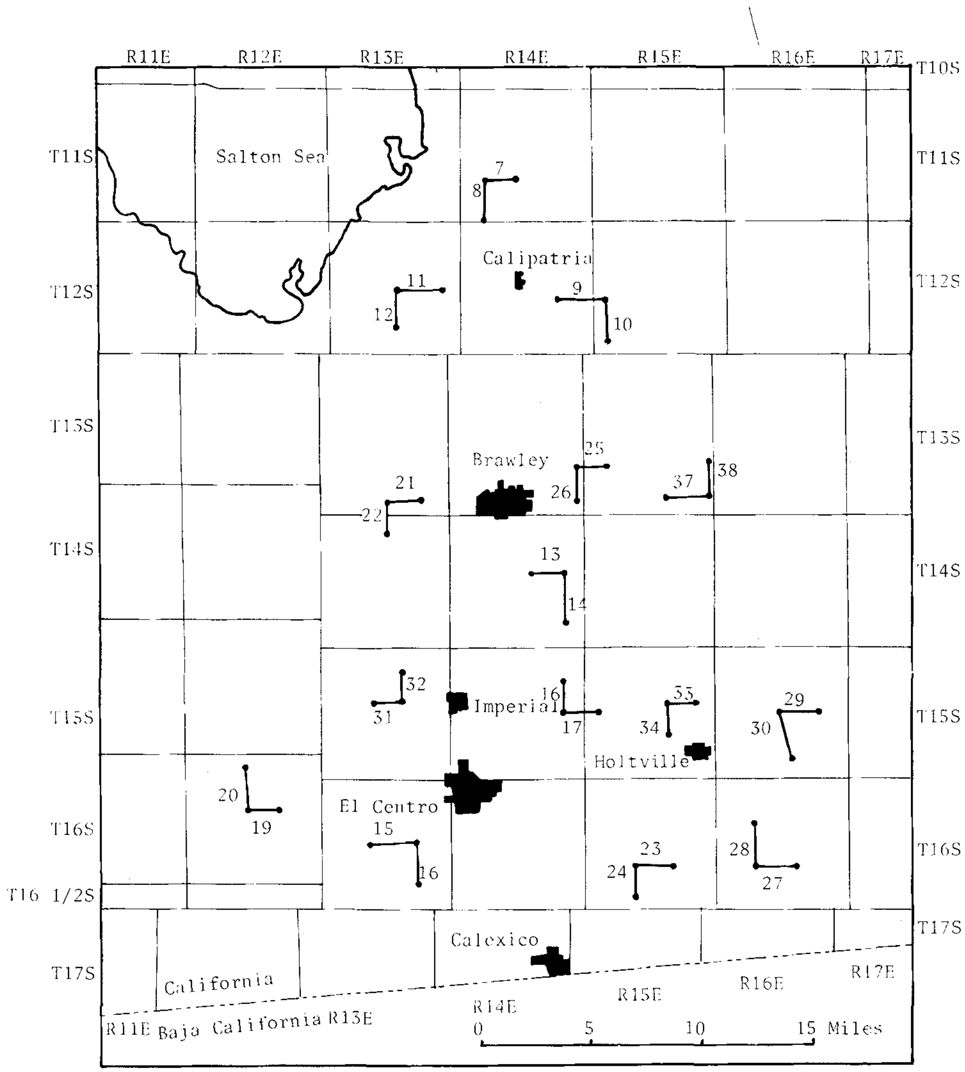


Figure 6.6A. *Locations of bipole sources used in a survey of the Imperial Valley, California.*

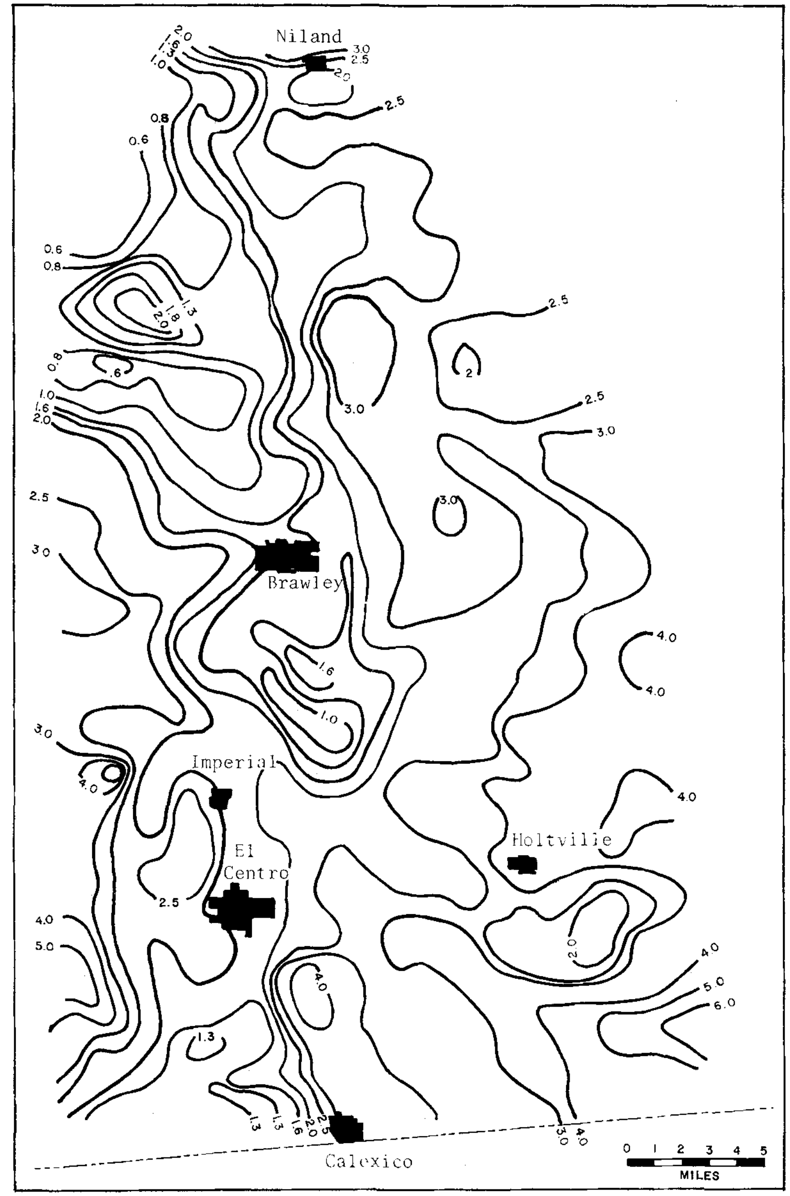


Figure 6.6B. *Spatially averaged total-field resistivity map of the Imperial Valley, California. Contours in ohm-m.*

Contents

has made evaluation of potential functions over a multilayer earth or any one-dimensional earth which can be represented by a Hankel transform a straight-forward matter even with small computers.

Three-Dimensional Forward Problem

Three-dimensional resistivity distributions are more difficult to handle for a variety of reasons. The primary reason is the large number of parameters which are necessary to describe a three-dimensional structure. If the structure is simple, as in the case of a dyke, a buried sphere or figure of revolution, straightforward analytical solutions can often be obtained. For example, Jain (1974) has presented curves for potential field behavior over simple dykes intersecting the earth's surface. Singh and Espindola (1976) and Scurta (1972) have examined the case of a single sphere buried in the earth. Bibby and Risk (1973) have given results for an ellipsoid of revolution which is bisected by the earth's surface. Each of these cases represents a body whose shape can be fitted to an equipotential surface in an orthogonal system of coordinates. Tabulation of such bodies and the appropriate coordinate systems has been given by Van Nostrand and Cook (1966). Another set of geometries which is amenable to direct solution is that of planar-perpendicular boundaries (Keller et al., 1975). Examples of the boundaries which may be considered are shown in Figure 6.7.

While many shapes of arbitrary bodies can be simulated with figures of revolution, the restrictions of the analytical approach leave one with the feeling that a large class of bodies cannot be handled (Hohman, 1975). Therefore,

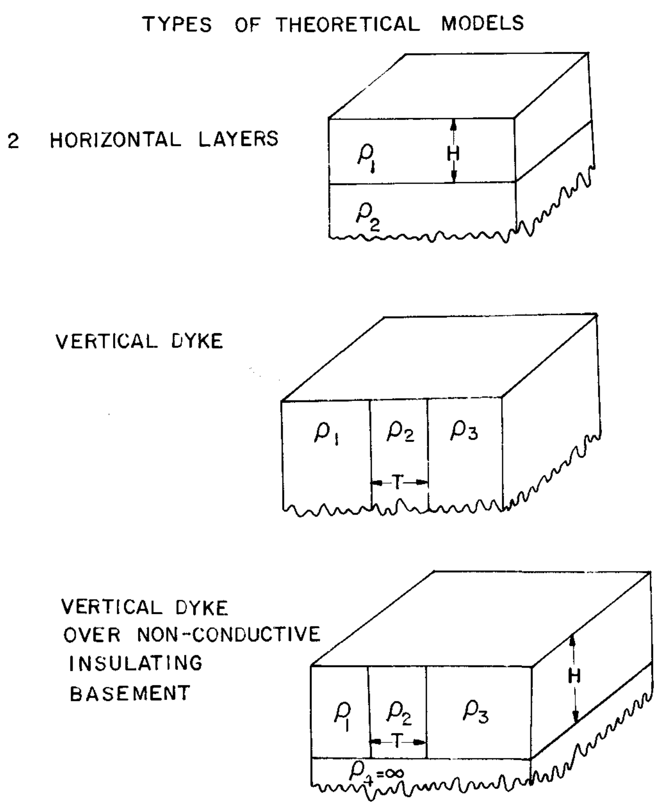


Figure 6.7. *Geometry of parallel-perpendicular boundaries that can be used in computing apparent resistivity.*

considerable effort has been spent in recent years in the solution of the problem of the potential distribution when current flows around and through a body of completely arbitrary shape. Basically, two approaches to the solution of this problem have appeared; the network approach, and the surface integration approach.

In the network approach, sometimes also known as the finite-element method or the finite-difference method, the earth is divided into a series of cells. Sometimes the cells are rectangular in shape, while in other cases the faces of the cells are triangular or cylindrical. At each point in the mesh representing that portion of the earth through which appreciable current flows, some relationship between current, potential, and structure of the earth is evaluated. For example, in the finite-difference method, the Laplacian equation is converted to a finite-difference equation where the second derivatives are calculated in terms of the differences in potential between adjacent points in the mesh. Then, at each point in the mesh, Laplace's equation can be evaluated in terms of the potentials at adjacent points in the mesh. A set of simultaneous equations which in number equals the number of mesh points nearly is developed. Solution of this system of equations yields values for the potential throughout the medium as well as at the surface where it is to be measured.

Other equivalent approaches may also be used. For example, the cells into which the earth is divided can be considered as resistance networks. The size of the resistor used to represent each cell is simply related to the size of the cell and its resistivity. At each point where the resistors are connected in the network representing the earth, continuity of current must be preserved. This provides the equation valid at that point. Simultaneous solution of equations for all points representing the medium again provides a numerical solution in terms of the voltage distribution throughout the model and at the surface. This approach has been used by Pires (1975) to provide an interpretation of bipole-dipole surveys carried out in northwestern Nevada and described in a previous part of this paper.

Still another condition which may be used to generate equations at each of the points in a mesh-represented earth is one used by Coggon (1971, 1973). He derived an expression for the energy loss for each element in a mesh representing the earth. He then applied the condition that the energy dissipated should be minimum for the actual flow of current in the earth. This provides a set of equations for solution which yields the potential at each point through the medium and on the surface.

Methods based on the use of meshes to represent the earth have not proved to be entirely satisfactory. They work best if the earth can be represented in two dimensions; that is, if the resistivity varies in one horizontal and one vertical direction. Then the cross section in apparent resistivity can be represented with reasonable detail by a small enough number of mesh points that a solution can be obtained on a reasonably large size computer. Mufti (1976) reduces the complexity of the two-dimensional problem by assuming infinite line electrodes, so that the problem can be specified in terms of a single cross section. Use of point electrodes leads to the need for three-dimensional representation of the medium, even though the earth structure is only two-dimensional. The complexity of this problem is reduced by recognizing that the field along the structure can be Fourier transformed so that parallel cross-sections of the earth can be treated sequentially rather than simultaneously (Stoyer, 1974; Coggon, 1971, 1973; Pelton et al., 1978). Axially symmetric bodies have been done by two-dimensional finite elements and Fourier transform by Bibby (1978).

Contents

When three-dimensional bodies are considered, the number of mesh points required to represent the body can be quite large. Bibby (1978) has shown how to handle axially symmetric bodies. However, to get a good representation of a general body, the number of equations which must be solved simultaneously exceeds the capability of even the large-scale computers currently available.

The surface integration method (Alfano, 1959; Keller and Frischknecht, 1966; Morozova, 1967; Dieter et al., 1969; Barnett, 1972; Snyder, 1976) is capable of representing a homogeneous three-dimensional body also. The surface integration method is based on the solution of the fundamental relationship between current and voltage in a resistive earth. Combination of the definition of electrostatic potential, U, in terms of electric field, E:

$$\vec{E} = -\text{grad } U$$

with the divergence condition for current flow, J

$$\text{div } \vec{J} = 0 \qquad \dots 6$$

and a relationship between electric field and current density

$$\vec{E} = \rho\vec{J} \qquad \dots 7$$

(where ρ is resistivity)

leads to an expression in the form of Poisson's equation representing the behavior of potential in the earth

$$\nabla^2 U = -\rho \left[\nabla \cdot \vec{J}_o + \nabla U \cdot \nabla\left(\frac{1}{\rho}\right)\right] \qquad \dots 8$$

where J_o is the source current system, which has potential U_o in a uniform region. The solution to Poisson's equation is well known:

$$U = U_o - \frac{1}{2\pi} \int_V \frac{\rho \nabla U \cdot \nabla\left(\frac{1}{\rho}\right)}{|R|} \, dV \qquad \dots 9$$

The first term in equation 9 represents the normal potential due to current flowing in a uniform earth with resistivity ρ. The second term represents the disturbing potential due to changes in resistivity within that earth. If changes take place at discrete boundaries, the second term is nonzero only at the boundaries. It can be considered conceptually as representing the effect of a series of current sources distributed over the surface of the arbitrary body. The current sources are of such strength as to match the boundary condition for current flow through that surface. The strength required for each of the elementary current sources over a surface is a function of the potential from all other current sources throughout the medium, both real and fictitious. Thus, a surface can be represented by some number, n, of current sources, and their strengths can be determined by a set of simultaneous equations. An example of the apparent resistivity curves over a buried body computed using this approach is shown in Figure 6.8.

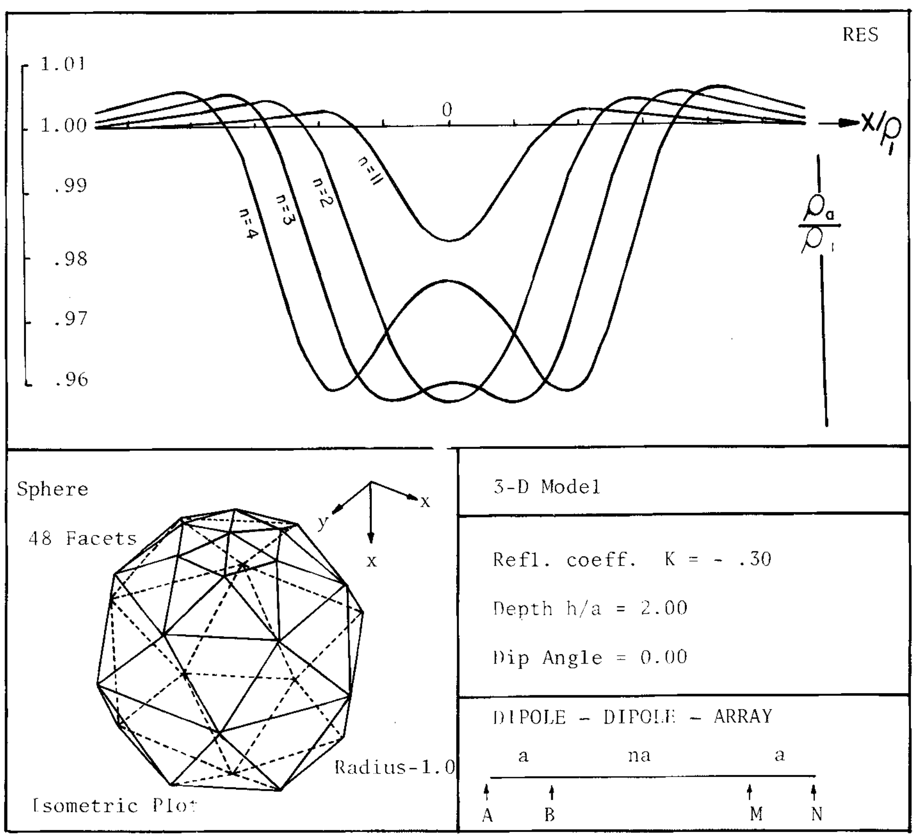


Figure 6.8. *Apparent resistivity profiles computed using surface integration for a buried, nearly spherical body (from Barnett, 1972).*

Inversion

In the preceding paragraphs, the various approaches which are available for determining the potential on the surface of the earth for virtually any conceivable structure of resistivity in the subsurface have been enunciated. With this capability, the concept of analytical inversion has appeared, and the technique has proved to be progressively more and more feasible. Various approaches to the inversion of field data to find the most likely resistivity distribution giving rise to those data have appeared (Parker, 1977).

The concept of linearization and solution of the inverse problem can be developed as follows. Following Crous (1971), the most frequently used criterion of how well two sets of points agree is the least squares criterion, defined as

$$E = \sum_{i=1}^{n} (\rho(x_i) - \hat{\rho}(x_i,P))^2 \qquad \dots 10$$

where $\rho(x_i)$ = observed apparent resistivities at points x_i

and $\hat{\rho}(x_i,P)$ = theoretical model apparent resistivities at points x_i when the model parameters are $P = (p_j)$, i j k

when the model parameters are changed from $\underline{P}$ to $\underline{P}+\Delta$

where $\Delta = (\delta_i)$

the theoretical apparent resistivities can be represented by a Taylor series

$$\hat{\rho}(x_i, \underline{P}+\Delta) = \hat{\rho}(x_i,\underline{P}) + \sum_{j=1}^{k} \left(\frac{\partial \rho}{\partial p_j}\right)_{x_i,\underline{P}} \delta_j + \dots \qquad \dots 11$$

where $\hat{\rho}(x_i,\hat{P}+\Delta)$ is the resistivity at points x_i for a set of parameters, $\hat{P}$, which describe the resistivity profile (layer thicknesses and resistivities) for the minimum error, and δ are displacements of each parameter from its ideal value for the initial model. Only the first derivative from the Taylor's expansion is given in equation 11; this corresponds to linearizing the problem. The mean-square error expressed in terms of equation 11 is minimized at each observation point. This provides a set of normal equations which must be solved to find the values for the parameters describing the earth that give the minimum error.

This set of equations can be solved only when each of the parameters of the earth model is uniquely related to the error. In many cases, this is not the case. Ambiguity may result from two or more parameters describing the resistivity in the earth affecting the error in a like manner or when the model has more parameters than there are independent observations. Therefore, before a solution can be obtained, the dependence of the error on each one of the parameters must be established to see that the parameters are mutually independent. If they are not, the parameters may be lumped in groups which are independent, or the problem may be modified to make the parameters independent. Details of the mathematical methods used in obtaining a solution to the set of normal equations have been described by Marquardt (1963), Backus and Gilbert (1967), Crous (1971), Inman et al. (1973), Inman (1975), Parker (1977), and Oldenburg (1978). Some examples of interpretation are given in Rijo et al. (1977) and Petrick et al. (1977).

Linearization is not necessary in finding the model which gives minimum error in interpretation. Another approach which is commonly used is that of the Fibinachi search. Here, some initial estimate of the interpretation is made. The error is computed in the least squares sense. Then, each of the parameters describing the resistivity structure of the earth is perturbed to find if the error is reduced. In this way, progressively better models are obtained to simulate the earth. If the perturbations follow a Fibinachi progression, it can be shown that the minimum is found with the least number of evaluations of the function, providing that the shape of the error function is at least convex. The assumption of linearity of the error is not required. However, if the initial guess is at all reasonable, the linearization can lead to a very rapid convergence on the minimum.

Still another approach to interpretation is sometimes called pseudo-inversion. Because the problems involved in inversion relate to the nonindependence of the parameters describing the model, difficulty can be avoided by fixing part of the parameters. One way of doing this is to assume that earth is made up of a sequence of layers of fixed thickness, usually thicknesses which increase in a geometric progression (Marsden, 1973). Then, solutions are sought only in terms of the resistivities for these preassigned layer thicknesses. Because ambiguity arises primarily from the interdependence of error between a resistivity and a thickness, most of the ambiguity in interpretation is removed by fixing the thickness.

Inversion techniques have proved highly effective for cases in which one-dimensional models are useful. An example of the comparison between field data and the expected curve for an interpretation obtained by inversion is shown in Figure 6.9. With a good set of field data, the RMS error between the curve calculated for the interpretation and

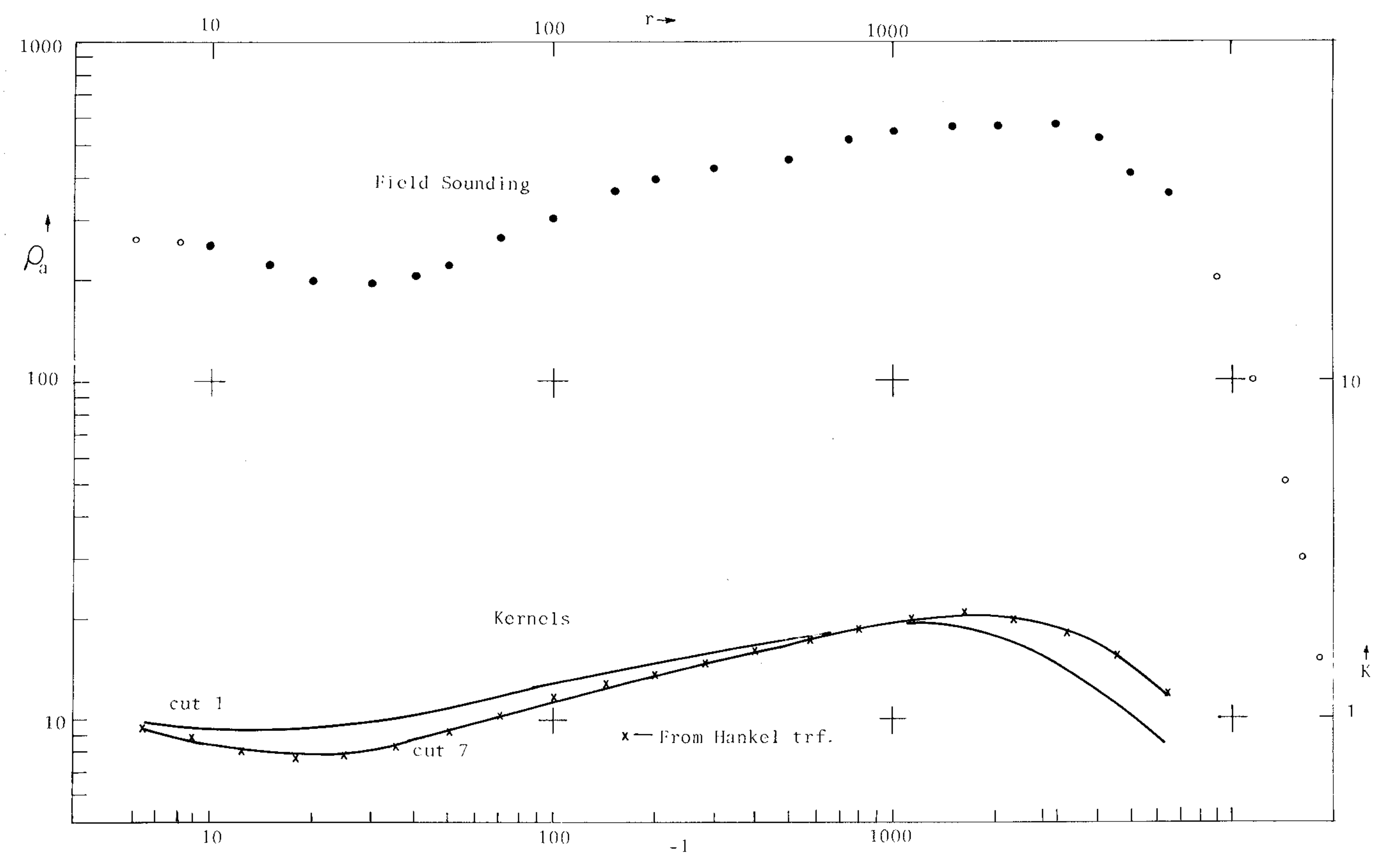


Figure 6.9. *Example of the inversion of a Schlumberger sounding curve. Cut 1 represents the initial estimate, while cut 7 reflects the fit to date after seven interations of an inversion scheme (Crous, 1971).*

of field observations can be expected to be less than one percent. To date, no convincing three-dimensional inversions have been reported in the literature. The difficulty with three-dimensional inversions arises because of the number of parameters that is necessary to describe the earth. Often, the number of parameters exceeds the number of observations so that the least squares process becomes indeterminate. Considerable research is being done at the present time on three-dimensional inversion schemes, so that some results may be expected in the near future.

SUMMARY AND CONCLUSIONS

While advances have been made in both the acquisition of field data in resistivity surveys and in the interpretation of those data, it appears that the capabilities of interpretive schemes has gone forward more rapidly than the capability for acquiring field data. It is to be expected that the capability to interpret field data will bring about some modifications in the way in which the field data are acquired to take advantage of these capabilities. It is my opinion that we can look forward to continued modification of field techniques, particularly in the direction of obtaining larger data sets from quantities of data to represent one particular survey to utilize in interpretation.

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