

# APPENDIX A. USE OF THE $M_{ia}$ , $M_{ib}$ , $M_{ih}$ , $M_{ic}$ IN PREDICTING COMMINUTION CIRCUIT SPECIFIC ENERGY

## A 1 INTRODUCTION

The following technical note describes the recently extended use of the SMC Test<sup>®</sup> to include crushers and HPGRs in determining the overall specific energy demand of comminution circuits. It builds on previous work which included tumbling mills only and should be read in conjunction with an earlier technical note dated September 2007 entitled “*Use of the SMC Test<sup>®</sup> in Predicting Total Comminution Circuit Specific Energy*” as well as published papers: Morrell, 2008a, 2008b. This enhancement now enables the SMC Test<sup>®</sup> to be used in conjunction with the Bond ball work index test to predict the specific energy of comminution circuits where such circuits include combinations of any of the following equipment:

- AG and SAG mills
- Ball mills
- Rod mills
- Crushers
- High pressure Grinding Rolls (HPGR)

## A 2 EQUATIONS

### A 2.1 General

The approach divides comminution equipment into three categories:

- Tumbling mills, eg AG, SAG, rod and ball mills
- Conventional reciprocating crushers, eg jaw, gyratory and cone
- HPGRs

Tumbling mills are described using 2 indices:  $M_{ia}$  and  $M_{ib}$

Crushers have one index:  $M_{ic}$

HPGRs have one index:  $M_{ih}$

For tumbling mills the 2 indices relate to “coarse” and “fine” ore properties plus an efficiency factor which represents the influence of a pebble crusher in AG/SAG mill circuits. “Coarse” in this case is defined as spanning the size range from a  $P_{80}$  of 750  $\mu\text{m}$  up to the  $P_{80}$  of the product of the last stage of crushing or HPGR size reduction prior to grinding. “Fine” covers the size range from a  $P_{80}$  of 750  $\mu\text{m}$  down to  $P_{80}$  sizes typically reached by conventional ball milling, ie about 45  $\mu\text{m}$ . The choice of 750  $\mu\text{m}$  as the division between “coarse” and “fine” particle sizes was determined during the development of the technique and was found to give the best overall results across the range of plants in SMCT’s database. Implicit in the approach is that distributions are parallel and linear in log-log space.

The work index covering grinding in tumbling mills of coarse sizes is labelled  $M_{ia}$ . The work index covering grinding of fine particles is labelled  $M_{ib}$ .  $M_{ia}$  values are provided as a standard output from a SMC Test<sup>®</sup> (Morrell, 2004<sup>a</sup>) whilst  $M_{ib}$  values can be determined using the data generated by a conventional Bond ball mill work index test ( $M_{ib}$  is NOT the Bond ball work index).  $M_{ic}$  and  $M_{ih}$  values are also provided as a standard output from a SMC Test<sup>®</sup>.

The general size reduction equation is as follows (Morrell, 2004<sup>b</sup>):

$$W_i = M_i 4 \left( x_2^{f(x_2)} - x_1^{f(x_1)} \right) \quad (1)$$

where

$M_i$  = Work index related to the breakage property of an ore (kWh/tonne). For grinding from the product of the final stage of crushing to a  $P_{80}$  of 750  $\mu\text{m}$  (coarse particles) the index is labelled  $M_{ia}$  and for size reduction from 750  $\mu\text{m}$  to the final product  $P_{80}$  normally reached by conventional ball mills (fine particles) it is labelled  $M_{ib}$ . For conventional crushing  $M_{ic}$  is used and for HPGRs  $M_{ih}$  is used.

$W_i$  = Specific comminution (kWh/tonne)

$x_2$  = 80% passing size for the product ( $\mu\text{m}$ )

$x_1$  = 80% passing size for the feed ( $\mu\text{m}$ )

$$f(x_j) = -(0.295 + x_j/1000000) \quad (\text{Morrell, 2006}) \quad (2)$$

For tumbling mills the specific comminution energy ( $W_i$ ) relates to the power at the pinion or for gearless drives - the motor output. For HPGRs it is the energy inputted to the rolls, whilst for conventional crushers  $W_i$  relates to the specific energy as determined using the motor input power less the no-load power.

### **A 2.2 Specific Energy Determination for Comminution Circuits**

The total specific energy ( $W_T$ ) to reduce in size primary crusher product to final product is given by:

$$W_T = W_a + W_b + W_c + W_h + W_s \quad (3)$$

where

$W_a$  = specific energy to grind coarser particles in tumbling mills

$W_b$  = specific energy to grind finer particles in tumbling mills

$W_c$  = specific energy for conventional crushing

$W_h$  = specific energy for HPGRs

$W_s$  = specific energy correction for size distribution

Clearly only the  $W$  values associated with the relevant equipment in the circuit being studied are included in equation 3.

#### **A 2.2.1 Tumbling mills**

For coarse particle grinding in tumbling mills equation 1 is written as:

$$W_a = K_1 M_{ia} 4 \left( x_2^{f(x_2)} - x_1^{f(x_1)} \right) \quad (4)$$

where

$K_1$  = 1.0 for all circuits that do not contain a recycle pebble crusher and 0.95 where circuits do have a pebble crusher

$x_1$  =  $P_{80}$  ( $\mu\text{m}$ ) of the product of the last stage of crushing before grinding

$x_2$  = 750  $\mu\text{m}$

$M_{ia}$  = Coarse ore work index and is provided directly by SMC Test<sup>®</sup>

For fine particle grinding equation 1 is written as:

$$W_b = M_{ib} 4 \left( x_3^{f(x_3)} - x_2^{f(x_2)} \right) \quad (5)$$

where

$x_2$  = 750  $\mu\text{m}$

$x_3$  =  $P_{80}$  ( $\mu\text{m}$ ) of final grind

$M_{ib}$  = Provided by data from the standard Bond ball work index test using the following equation (Morrell, 2006):

$$M_{ib} = \frac{18.18}{P_1^{0.295} (Gbp) \left( p_{80}^{f(p_{80})} - f_{80}^{f(f_{80})} \right)} \quad (6)$$

where

$M_{ib}$  = fine ore work index (kWh/tonne)

$P_1$  = closing screen size ( $\mu\text{m}$ )

$Gbp$  = net grams of screen undersize per mill revolution

$p_{80}$  = 80% passing size of the product ( $\mu\text{m}$ )

$f_{80}$  = 80% passing size of the feed ( $\mu\text{m}$ )

Note that the Bond ball work index test should be carried out with a closing screen size chosen to give a final product  $P_{80}$  similar to that intended for the full-scale circuit.

### A 2.2.2 Conventional Crushers

Equation 1 for conventional crushers is written as:

$$W_c = K_2 M_{ic} 4 \left( x_2^{f(x_2)} - x_1^{f(x_1)} \right) \quad (7)$$

$K_2$  = 1.0 for all crushers operating in closed circuit with a classifying screen. If the crusher is in open circuit, eg pebble crusher in a AG/SAG circuit,  $K_2$  takes the value of 1.19.

$x_1$  =  $P_{80}$  ( $\mu\text{m}$ ) of the circuit feed

$x_2$  =  $P_{80}$  ( $\mu\text{m}$ ) of the circuit product

$M_{ic}$  = Crushing ore work index and is provided directly by SMC Test<sup>®</sup>

### A 2.2.3 HPGR

Equation 1 for HPGR crushers is written as:

$$W_h = K_3 M_{ih} 4 \left( x_2^{f(x_2)} - x_1^{f(x_1)} \right) \quad (8)$$

$K_3$  = 1.0 for all HPGRs operating in closed circuit with a classifying screen. If the HPGR is in open circuit,  $K_3$  takes the value of 1.19.

$x_1$  =  $P_{80}$  ( $\mu\text{m}$ ) of the circuit feed

$x_2$  =  $P_{80}$  ( $\mu\text{m}$ ) of the circuit product

$M_{ih}$  = HPGR ore work index and is provided directly by SMC Test<sup>®</sup>

### A 2.2.4 Specific Energy Correction for Size Distribution ( $W_s$ )

Implicit in the approach described in this paper is that the feed and product size distributions are parallel and linear in log-log space. Where they are not, allowances (corrections) need to be made. By and large, such corrections are most likely to be necessary (or are large enough to be warranted) when evaluating circuits in which closed circuit secondary/tertiary crushing is followed by ball milling. This is because such crushing circuits tend to produce a product size distribution which is relatively steep when compared to the ball mill circuit cyclone overflow. This is illustrated in App Figure 1, which shows measured distributions from an open and closed crusher circuit as well as a ball mill cyclone overflow. The closed circuit crusher distribution

can be seen to be relatively steep compared with the open circuit crusher distribution and ball mill cyclone overflow. Also the open circuit distribution more closely follows the gradient of the cyclone overflow. If a ball mill circuit were to be fed 2 distributions, each with same  $P_{80}$  but with the open and closed circuit gradients in App Figure 1, the closed circuit distribution would require more energy to grind to the final  $P_{80}$ . How much more energy is required is difficult to determine. However, for the purposes of this approach it has been assumed that the additional specific energy for ball milling is the same as the difference in specific energy between open and closed crushing to reach the nominated ball mill feed size. This assumes that a crusher would provide this energy. However, in this situation the ball mill has to supply this energy and it has a different (higher) work index than the crusher (ie the ball mill is less energy efficient than a crusher and has to input more energy to do the same amount of size reduction). Hence from equation 7, to crush to the ball mill circuit feed size ( $x_2$ ) in open circuit requires specific energy equivalent to:

$$W_c = 1.19 * M_{ic} 4 \left( x_2^{f(x_2)} - x_1^{f(x_1)} \right) \quad (9)$$

For closed circuit crushing the specific energy is:

$$W_c = 1 * M_{ic} 4 \left( x_2^{f(x_2)} - x_1^{f(x_1)} \right) \quad (10)$$

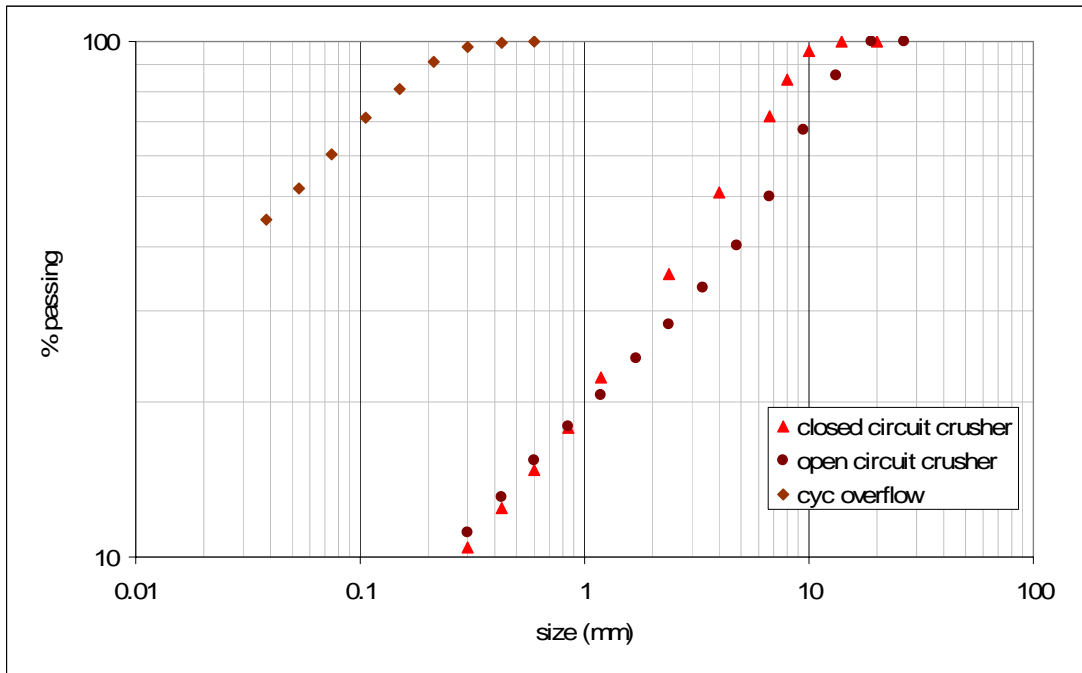
The difference between the two (eq 9 – eq 10) has to be provided by the milling circuit with an allowance for the fact that the ball mill, with its lower energy efficiency, has to provide it and not the crusher. This is what is referred to in equation 3 as  $W_s$  and for the above example is therefore represented by:

$$W_s = 0.19 * M_{ia} 4 \left( x_2^{f(x_2)} - x_1^{f(x_1)} \right) \quad (11)$$

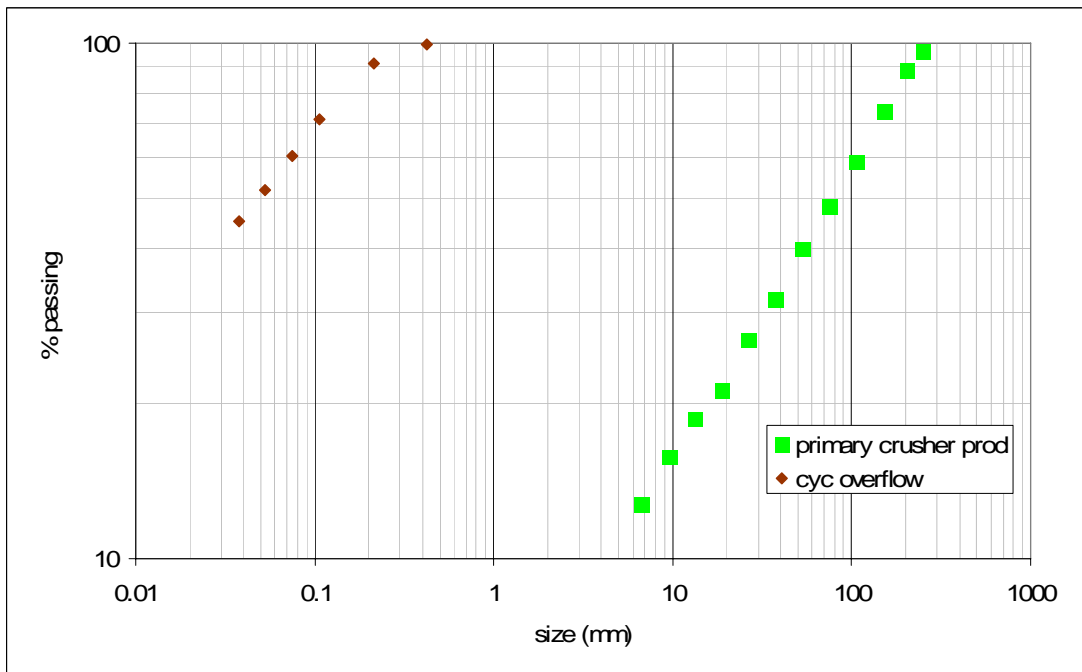
Note that in equation 11  $M_{ic}$  has been replaced with  $M_{ia}$ , the coarse particle tumbling mill grinding work index.

In AG/SAG based circuits the need for  $W_s$  appears to be unnecessary as App Figure 2 illustrates. Primary crusher feeds often have the shape shown in App Figure 2 and this has a very similar gradient to typical ball mill cyclone overflows. A similar situation appears to apply with HPGR product size distributions, as illustrated in App Figure 3. Interestingly SMCT's data show that for HPGRs, closed circuit operation appears to require a lower specific energy to reach the same  $P_{80}$  as in open circuit, even though the distributions for open and closed circuit look to have almost identical gradients. Closer examination of the distributions in fact shows that in closed circuit the final product tends to have slightly less very fine material, which may account for the different energy requirements between the two modes of operation. It is also possible that recycled material in closed circuit is inherently weaker than new feed, as it has already passed through the HPGR previously and may have sustained micro-cracking. A reduction in the Bond ball mill work index as measured by testing HPGR products compared it to the Bond ball mill work index of HPGR feed has been noticed in many cases in the laboratory (see next section) and hence there is no reason to expect the same phenomenon would not affect the recycled HPGR screen oversize.

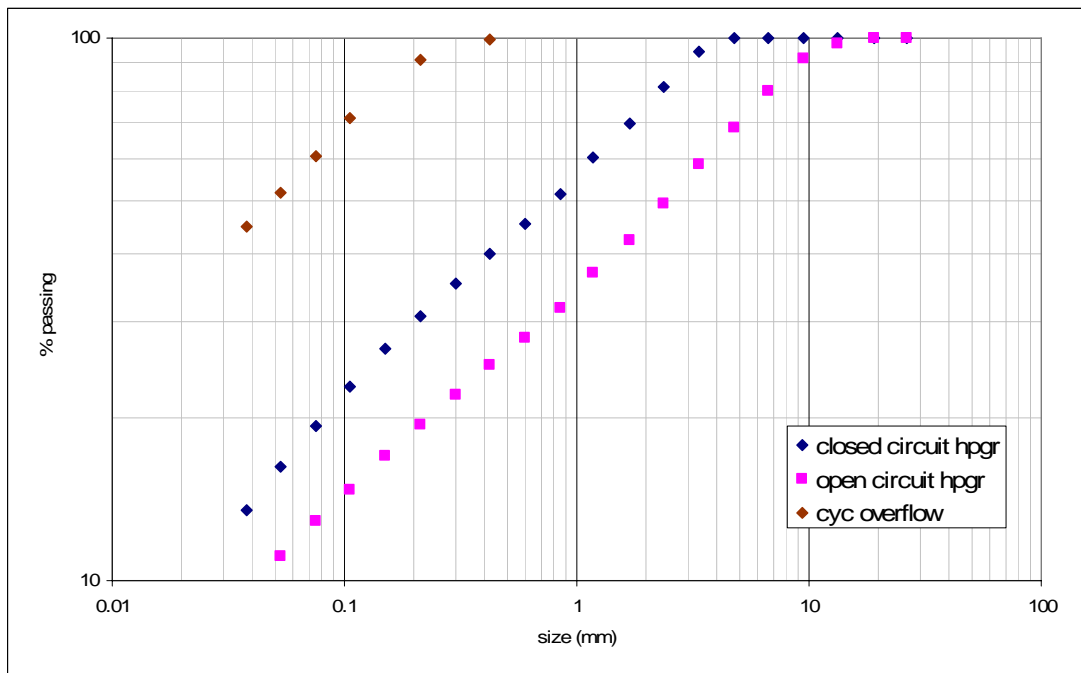
It follows from the above arguments that in HPGR circuits, which are typically fed with material from closed circuit secondary crushers, a similar feed size distribution correction should also be applied. However, as the secondary crushing circuit uses such a relatively small amount of energy compared to the rest of the circuit (as it crushes to a relatively coarse size) the magnitude of size distribution correction is very small indeed – much smaller than the error associated with the technique - and hence may be omitted in calculations.



App Figure 1 – Examples of Open and Closed Circuit Crushing Distributions Compared with a Typical Ball Mill Cyclone Overflow Distribution



App Figure 2 – Example of a Typical Primary Crusher (Open and Circuit) Product Distribution Compared with a Typical Ball Mill Cyclone Overflow Distribution



App Figure 3 – Examples of Open and Closed Circuit HPGR Distributions Compared with a Typical Ball Mill Cyclone Overflow Distribution

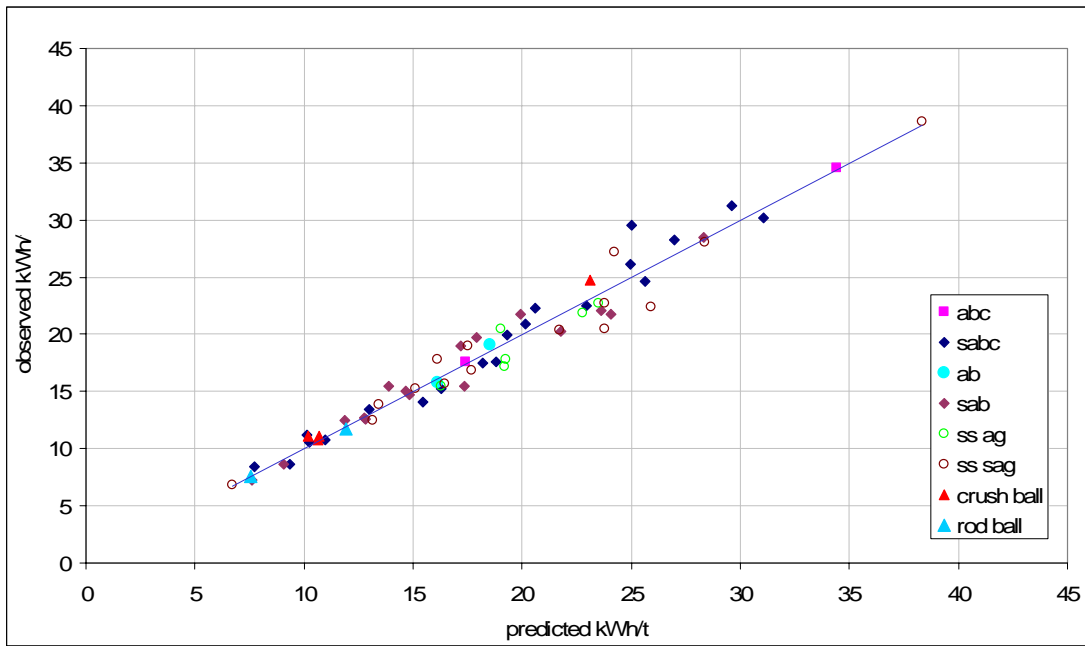
### A 2.2.5 Weakening of HPGR Products

As mentioned in the previous section, laboratory experiments have been reported by various researchers in which the Bond ball work index of HPGR products is less than that of the feed. The amount of this reduction appears to vary with both material type and the pressing force used. Observed reductions in the Bond ball work index have typically been in the range 0-10%. In the approach described in this paper no allowance has been made for such weakening. However, if HPGR products are available which can be used to conduct Bond ball work index tests on then  $M_{ib}$  values obtained from such tests can be used in equation 5. Alternatively the  $M_{ib}$  values from Bond ball mill work index tests on HPGR feed material can be reduced by an amount that the user thinks is appropriate. Until more data become available from full scale HPGR/ball mill circuits it is suggested that, in the absence of Bond ball mill work index data on HPGR products, the  $M_{ib}$  results from HPGR feed material are reduced by no more than 5% to allow for the effects of micro-cracking.

## A 3 VALIDATION

### A 3.1 *Tumbling Mill Circuits*

The approach described in the previous section was applied to 65 industrial data sets. The results are shown in App Figure 4. In all cases the specific energy relates to the tumbling mills contributing to size reduction from the product of the final stage of crushing to the final grind. Data are presented in terms of equivalent specific energy at the pinion. In determining what these values were on each of the plants in the data base it was assumed that power at the pinion was 93.5% of the measured gross (motor input) power, this figure being typical of what is normally accepted as being reasonable to represent losses across the motor and gearbox. For gearless drives (so-called wrap-around motors) a figure of 97% was used.



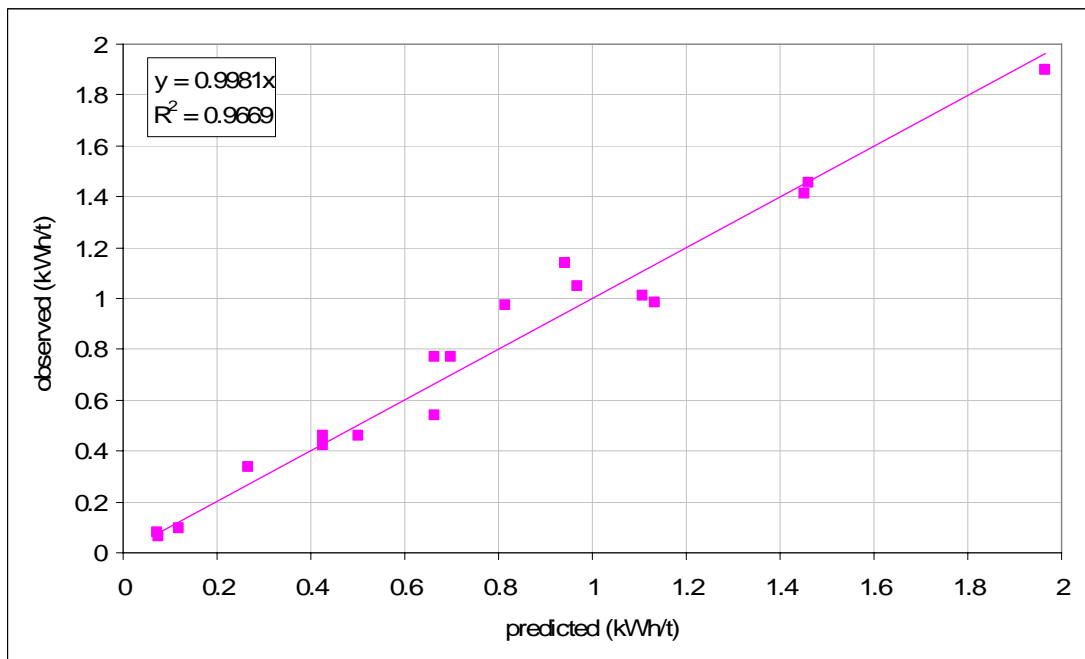
App Figure 4 – Observed vs Predicted Tumbling Mill Specific Energy

### A 3.2 Conventional Crushers

Validation of equation 1 used 10 different crushing circuits (18 data sets), including secondary, tertiary and pebble crushers in AG/SAG circuits. Observed vs predicted specific energies are given in App Figure 5. The observed specific energies were calculated from the crusher throughput and the net power draw of the crusher as defined by:

$$\text{Net Power} = \text{Motor Input Power} - \text{No Load Power} \quad (12)$$

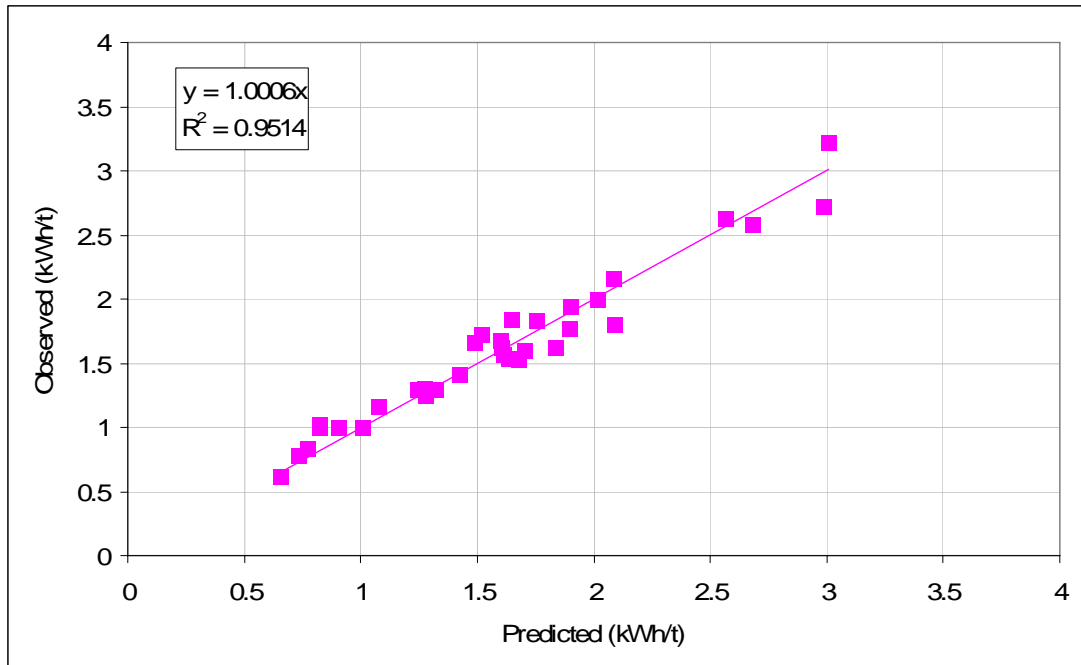
No-load power tends to be relatively high in conventional crushers and hence net power is significantly lower than the motor input power. From examination of the 18 crusher data sets the motor input power was found to be on average 35% higher than the net power.



App Figure 5 – Observed vs Predicted Conventional Crusher Specific Energy

### A 3.3 HPGRs

Validation of equation 1 for HPGRs used data from 18 different circuits (35 data sets) including laboratory, pilot and industrial scale equipment. Observed vs predicted specific energies are given in App Figure 6. The data relate to HPGRs operating with specific grinding forces typically in the range 2.5-3.5 N/mm<sup>2</sup>. The observed specific energies relate to power delivered by the roll drive shafts. Motor input power for full scale machines is expected to be 8-10% higher.



App Figure 6 – Observed vs Predicted HPGR Specific Energy

### A 4 WORKED EXAMPLES

A SMC Test<sup>®</sup> and Bond ball work index test were carried out on a representative ore sample. The following results were obtained:

SMC Test:

$$M_{ia} = 19.4 \text{ kWh/t}$$

$$M_{ic} = 7.2 \text{ kWh/t}$$

$$M_{ih} = 13.9 \text{ kWh/t}$$

Bond test carried out with a 150 micron closing screen:

$$M_{ib} = 18.8 \text{ kWh/t}$$

Three circuits are to be evaluated:

- SABC
- HPGR/ball mill
- Conventional crushing/ball mill

The overall specific grinding energy to reduce a primary crusher product with a  $P_{80}$  of 100 mm to a final product  $P_{80}$  of 106  $\mu\text{m}$  needs to be estimated.

#### A 4.1 SABC Circuit

Coarse particle tumbling mill specific energy

Combining eq 2 and 4:



$$W_a = 0.95 * 19.4 * 4 * \left( 750^{-\left(\frac{0.295+750}{1000000}\right)} - 100000^{-\left(\frac{0.295+100000}{1000000}\right)} \right)$$

$$= 9.6 \text{ kWh/t}$$

#### Fine particle tumbling mill specific energy

Combining eq 2 and 5:

$$W_b = 18.8 * 4 * \left( 106^{-\left(\frac{0.295+106}{1000000}\right)} - 750^{-\left(\frac{0.295+750}{1000000}\right)} \right)$$

$$= 8.4 \text{ kWh/t}$$

#### Pebble crusher specific energy

In this circuit, it is assumed that the pebble crusher feed  $P_{80}$  is 52.5mm. As a rule of thumb this value can be estimated by assuming that it is 0.75 of the nominal pebble port aperture (in this case the pebble port aperture is 70mm). The pebble crusher is set to give a product  $P_{80}$  of 12mm. The pebble crusher feed rate is expected to be 25% of new feed tph.

Combining eq 2 and 7:

$$W_c = 1.19 * 7.2 * 4 * \left( 12000^{-\left(\frac{0.295+12000}{1000000}\right)} - 52500^{-\left(\frac{0.295+52500}{1000000}\right)} \right)$$

$$= 1.12 \text{ kWh/t when expressed in terms of the crusher feed rate}$$

$$= 1.12 * 0.25 \text{ kWh/t when expressed in terms of the SABC circuit new feed rate}$$

$$= 0.3 \text{ kWh/t of SAG mill circuit new feed}$$

#### Total net comminution specific energy:

From eq 3:

$$W_T = 9.6 + 8.4 + 0.3 \quad \text{kWh/t}$$

$$= 18.3 \text{ kWh/t}$$

#### **A 4.2 HPGR/Ball Milling Circuit**

In this circuit primary crusher product is reduced to a HPGR circuit feed  $P_{80}$  of 35 mm by closed circuit secondary crushing. The HPGR is also in closed circuit and reduces the 35 mm feed to a circuit product  $P_{80}$  of 4 mm. This is then fed to a closed circuit ball mill which takes the grind down to a  $P_{80}$  of 106  $\mu\text{m}$ .

#### Secondary crushing specific energy

Combining eq 2 and 7:

$$W_c = 1 * 7.2 * 4 * \left( 35000^{-\left(\frac{0.295+35000}{1000000}\right)} - 100000^{-\left(\frac{0.295+100000}{1000000}\right)} \right)$$

$$= 0.6 \text{ kWh/t}$$

#### HPGR specific energy

Combining eq 2 and 8:

$$W_c = 1 * 13.9 * 4 * \left( 4000^{-\left(\frac{0.295+4000}{1000000}\right)} - 35000^{-\left(\frac{0.295+35000}{1000000}\right)} \right)$$

$$= 2.9 \text{ kWh/t}$$

#### Coarse particle tumbling mill specific energy

Combining eq 2 and 4:

$$W_a = 1 * 19.4 * 4 * \left( 750^{-\frac{0.295+750}{1000000}} - 4000^{-\frac{0.295+4000}{1000000}} \right)$$

$$= 4.5 \text{ kWh/t}$$

#### Fine particle tumbling mill specific energy

Combining eq 2 and 5:

$$W_b = 18.8 * 4 * \left( 106^{-\frac{0.295+106}{1000000}} - 750^{-\frac{0.295+750}{1000000}} \right)$$

$$= 8.4 \text{ kWh/t}$$

#### Total net comminution specific energy:

From eq 3:

$$W_T = 4.5 + 8.4 + 0.6 + 2.9 \quad \text{kWh/t}$$

$$= 16.4 \text{ kWh/t}$$

### **A 4.3 Conventional Crushing/Ball Milling Circuit**

In this circuit primary crusher product is reduced in size to P<sub>80</sub> of 6.5 mm via a secondary/tertiary crushing circuit (closed). This is then fed to a closed circuit ball mill which grinds to a P80 of 106 μm.

#### Secondary/tertiary crushing specific energy

Combining eq 2 and 7:

$$W_c = 1 * 7.2 * 4 * \left( 6500^{-\frac{0.295+6500}{1000000}} - 100000^{-\frac{0.295+100000}{1000000}} \right)$$

$$= 1.7 \text{ kWh/t}$$

#### Coarse particle tumbling mill specific energy

Combining eq 2 and 4:

$$W_a = 1 * 19.4 * 4 * \left( 750^{-\frac{0.295+750}{1000000}} - 6500^{-\frac{0.295+6500}{1000000}} \right)$$

$$= 5.5 \text{ kWh/t}$$

#### Fine particle tumbling mill specific energy

Combining eq 2 and 5:

$$W_b = 18.8 * 4 * \left( 106^{-\frac{0.295+106}{1000000}} - 750^{-\frac{0.295+750}{1000000}} \right)$$

$$= 8.4 \text{ kWh/t}$$

#### Size distribution correction

$$W_s = 0.19 * 19.4 * 4 * \left( 6500^{-\frac{0.295+6500}{1000000}} - 100000^{-\frac{0.295+100000}{1000000}} \right)$$

$$= 0.9 \text{ kWh/t}$$

#### Total net comminution specific energy:

From eq 3:

$$W_T = 5.5 + 8.4 + 1.7 + 0.9 \text{ kWh/t}$$

$$= 16.5 \text{ kWh/t}$$